Bell's Theorem and Einstein's 'Spooky Actions' from a Simple Thought Experiment[1]

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In 1964 John Bell proved a theorem[2] allowing the experimental test of whether what Einstein derided as "spooky actions at a distance" actually exist. We will see that they *do*. Bell's theorem can be displayed with a simple, nonmathematical thought experiment suitable for a physics course at *any* level. And a simple, semi-classical derivation of the quantum theory result can be given for physics students. These entanglement phenomena are today applied in industrial laboratories and are increasingly discussed in the popular literature. Unfortunately, they are also misappropriated by the purveyors of pseudoscience, something physicists have a responsibility to address.[3] Students can be intrigued by the quantum strangeness physics has encountered at a boundary of our discipline.

The quantum enigma

The Bell's theorem story starts with Einstein's early objections to the unreal world quantum theory apparently described. Bohr had embarrassed Einstein at two conferences by refuting his challenges to the theory. Bohr then assumed that a humbled Einstein went home to concentrate on general relativity. Bohr was wrong. Four years later, in 1935, a paper[4] by Einstein and two young colleagues, Boris Podolsky and Nathan Rosen, arrived in Copenhagen. It is now famous as "EPR." A colleague of Bohr writes,"This onslaught came down upon us like a bolt from the blue."

The Copenhagen interpretation of quantum mechanics, physics' "orthodox position," holds that "[o]bservations not only *disturb* what is to be measured, they *produce* it."[5] If observations *produce* the observed physical properties, were those physical properties "physically real" *before* their observation? The meaning of "physical reality" has been debated for millennia, and is still debated. In this paper we will assume the EPR definition of physical reality: "If without in any way disturbing [or observing] a system, we can predict with certainty … the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

EPR showed that you could indeed *know* a property of an object *without* that property being observed. It must therefore have existed *prior* to its observation as an "element of physical reality." EPR argued that quantum theory was incomplete because it did not include such supposedly existing real properties, later called "hidden variables."

The EPR paper involved an ingenious but hard-to-visualize mathematical trick. David Bohm's easier-to-describe version of EPR[6] is also basic to the Bell's theorem experiments. Consider two photons emitted in a rapid cascade by a single atom and traveling in opposite directions. If the net angular momentum change of the emitting atom is zero, the pair of photons must carry off zero net angular momentum. Such photons are "entangled," in a "twin state," with identical linear polarizations. They will always behave as if they had the *same* polarization.

Since the identical linear polarization properties of such twin-state photons can be verified experimentally, explaining their creation is not crucial to the presentation of Bell's theorem. But the explanation is straightforward. For example, consider an atom falling from a state with $J = 0$ to a state with $J = 1$, emitting the first photon, and then rapidly falling to a state with $J = 0$, emitting the second photon. Since the angular momentum of the atom was finally unchanged, the twin state *pair* of emitted photons carry off zero *net* angular momentum.[7] But each photon, being emitted in a $\Delta J = \pm 1$ transition, must carry off one unit of angular momentum. Moving in opposite directions, and having zero *total* angular momentum, the photons must both exhibit right-handed circular polarization or both exhibit left-handed circular polarization. This is mathematically equivalent to the two photons having the same linear polarization.[8]

Although photons in a twin state have the *same* polarization, quantum theory tells us that neither photon has a *particular* polarization until its polarization is observed. Since the two photons separate at twice the speed of light, nothing at the site of one should be able to physically affect its twin. EPR assumes this "separability." Nevertheless, according to quantum theory, observing the polarization of one photon determines with certainty the polarization of its remote twin. But the polarization of that remote photon may as yet be *unobserved*. Therefore its polarization was not *created* by observation. That polarization must therefore, according to EPR, exist within that photon as a physical reality, a "hidden variable."

Bohr responded to EPR by agreeing that no physical force could connect the observations of the two photons. Nevertheless, he maintained, the polarization one experimenter observed "influenced" what would be seen by the observer of the photon's twin. Einstein rejected what Bohr called "influences" as "spooky actions at a distance."

Bohr and Einstein agreed on the *results* of any EPR experiment. They just disagreed on the meaning. Are there hidden variables? Or do Bohr's "influences" actually exist? For three decades after EPR, most physicists considered these to be

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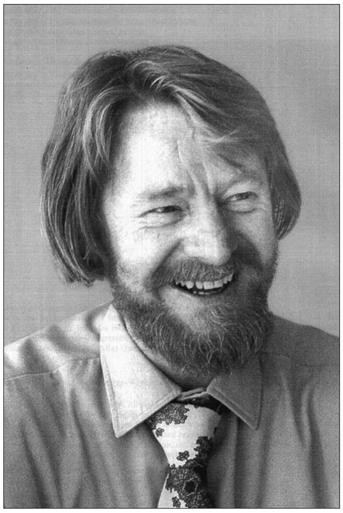

Fig. 1. John Stewart Bell

unanswerable philosophical questions, therefore pointless. Bell's theorem changed that.

Bell's theorem led to experiments that answered these "philosophical" questions in the laboratory: Objects that have ever interacted forever *do* influence each other instantaneously. The existence of Bohr's "influences," Einstein's "spooky actions," has now been established. But they are no less spooky.

After either object has interacted significantly with the environment, which happens extremely rapidly for macroscopic objects, these influences are undetectable for all practical purposes. Nevertheless, these quantum influences get attention today in industrial laboratories because they may make possible fantastically powerful computers. They are already used to encrypt communications.

John Bell was born in Belfast in 1928. Though no one in the family had ever had even a secondary school education, his mother promoted learning as the way to the good life, in which you "could wear Sunday suits all week." Eager for knowledge, Bell spent time in the library instead of going off with the other boys, which he would have done had he been, he says, "more gregarious, more socially adequate."

Early on, philosophy attracted Bell. But he moved to physics, where "you could reasonably come to conclusions." At Queen's, the local university, Bell felt the quantum mechanics courses concentrated too strongly on the practical aspects of the theory and too little on its deeper meaning. Nevertheless, he went to work in an almost engineering role, the design of particle accelerators. He married a fellow physicist, Mary Ross. Though they worked independently, Bell writes that in looking through his collected papers, "I see her everywhere."

At CERN (the European Center for Nuclear Research) Bell concentrated on the mainstream physics that he felt he was paid to do, and of which his colleagues approved. He restrained his interest in the strangeness of quantum mechanics for years. "[Sabbatical leave in 1964], away from the people who knew me, gave me more freedom, so I spent some time on these quantum questions." The momentous result is what we now call "Bell's theorem."

In 1989, at a small conference in Erice, Sicily, that focused on his work, Bell emphasized, with wit and in his Irish voice, the depth of the quantum enigma. In big bold letters on the blackboard he introduced his famous abbreviation, FAPP ("for all practical purposes") and warned against falling into the FAPPTRAP: accepting a merely FAPP solution for the enigma. The next year John Bell suddenly died.

Bell's theorem

Bell's theorem in a nutshell: Suppose that unobserved objects in our world have physically real properties that are *not* created by their observation. Further suppose that two objects can be separated from each other so that what happens to one cannot instantaneously affect the other. For short, we'll call these two suppositions "reality" and "separability." From these two premises, both *denied* by quantum theory, Bell deduced that certain observable quantities *could not be larger* than certain other observable quantities. This *experimentally testable* prediction of Bell's theorem is "Bell's inequality."

We won't go for suspense. When the experiments were done, Bell's inequality *was* (as Bell expected) violated. The assumptions of reality and separability yield a *wrong* prediction. We therefore know that our world does *not* have both reality and separability.

Derivation of a Bell inequality

We will demonstrate Bell's theorem with something *like* twin-state photons. By assuming Bell's premises of reality and separability, we will end up with a Bell inequality. Specifically, we will assume that each of our "twin-state photons" has a real "polarization angle" (a hidden variable) that is not created by its observation. And we assume that the two photons are separable, that what happens to one cannot affect its twin.

Assuming reality and separability, we are assuming things that quantum theory *denies*. What we will here call "photons" are therefore *not* like the photons described by quantum theory. Are they like the photons that make Geiger counters click in our actual world? That's what *actual experiments* must decide. We'll refer to them as "photons," keeping the quote marks.

To be concrete, we'll present a specific mechanical picture. However, the logic we use *in no way depends on any aspect of this mechanical model* except its reality and separability. Bell's mathematical treatment was completely general. It did not even specify photons.

Our assumptions of reality and separability will lead to a testable prediction. Since that prediction will be seen to be *wrong* in actual experiments, at least one of the assumptions leading to that prediction must be wrong. Our actual world therefore *cannot* have both reality and separability. Nick Herbert invented the general idea we use.[9]

To show a "photon's" assumed polarization as *graphically* real, we show each "photon" as a "stick" in our figures. The angle of the stick is our "photon's" polarization, a hidden variable. A polarizer in this mechanical model is a plate with an oval opening whose long dimension is the "polarizer axis." A "photon" whose polarization direction is close to the polarizer axis will pass through the polarizer to go on Path 1. One whose polarization is not close will hit the polarizer to fall on Path 2.

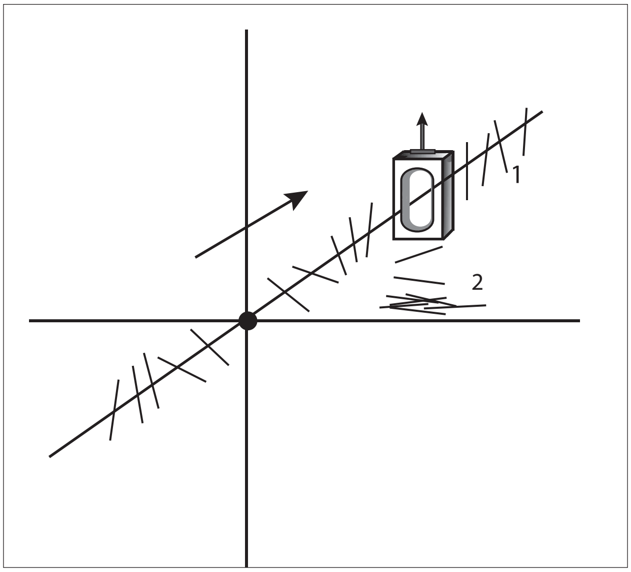

Fig. 2. Model of stick photons and oval polarizer.

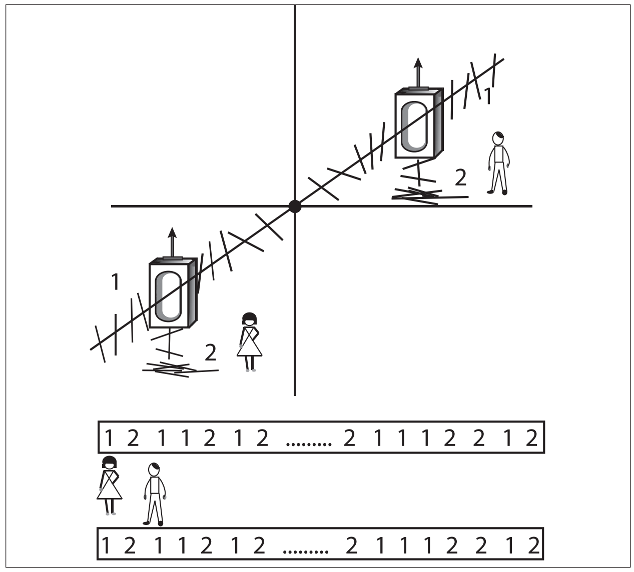

Fig. 3. Experiment I. Polarizers aligned. Alice's and Bob's data identical.

A "photon's" behavior at a polarizer is determined by a physically real property of that "photon," the orientation of its stick; that's our reality assumption. A "photon's" behavior is not affected by that of its twin; that's our separability assumption. This mechanical model does not account properly for all the behavior of polarized light. But that does not matter; our logic ultimately depends on nothing about these "photons" except their reality and separability.

We will describe four Alice-and-Bob thought experiments. These are much like the thought experiment proposed by EPR, but there is a big difference: Bohr and Einstein agreed on the results predicted for an EPR experiment. They would differ only on the *interpretation* of those results. In our model, and in the actual Bell's theorem experiments, both the predictions and the actual experimental results for Einstein's "hidden variables" and Bohr's "influences" are different.

In each of our four experiments, twin-state "photons" with identical polarizations (identical stick angles) are emitted in opposite directions from a source between Alice and Bob, a bit closer to Alice. Since the "photons" fly apart from each other at the speed of light, which no physical object can exceed, nothing physical can get from Alice to Bob before his "photon" arrives at his polarizer. Alice and Bob identify "photons" as being twins by their almost identical arrival times and keep track of whether their Path 1 or Path 2 detector recorded each "photon."

Experiment I:

Alice and Bob each have their polarizer axes aligned vertically. They record a "1" every time their Path 1 detector records a "photon" and a "2" every time their Path 2 detector records one. They each end up with a string of random 1s and 2s.

After recording a large number of "photons," Alice and Bob come together and compare their results. They find their data streams identical. Bob's "photon" took the same path at his polarizer as its twin did at Alice's. This confirms that their almost simultaneously arriving "photons" were indeed twins. Even before Bob joined Alice, Alice could predict with certainty the polarization of each of Bob's "photons," even *before* they arrived at Bob's polarizer. It's the EPR result.

Alice and Bob expected this perfect matching. Their twin "photons" indeed *had* identical polarization, identical stick angles. (In quantum theory, where a polarization direction is observer-created, the matching must be explained by what Bohr called an instantaneous "influence" on a photon by the observation of its twin.)

Experiment II:

This is the same as Experiment I, except this time Alice rotates her polarizer by a small angle θ. Bob keeps his polarizer axis vertical.

By our reality assumption, the polarization angle of these "photons" is unaffected by Alice's observation or her polarizer axis. Therefore some "photons" that would have gone through Alice's polarizer on her Path 1, had she not rotated it, now go on her Path 2, and vice versa. By our separability assumption, Bob's "photons" are unaffected by Alice's polarizer rotation or by what happened to their twins at Alice's polarizer.

Alice and Bob, coming together this time to compare their data streams, find some mismatches. Mismatches arise because when some of Alice's "photons" went on her Path 2, their twins at Bob's polarizer went not on his Path 2 but on his Path 1, and vice versa. The percentage of mismatches would be small for small θ. Let's say that Alice changed what would have happened for 5% of her "photons." She thus caused a mismatch rate of 5%.

Experiment III:

This is exactly the same as Experiment II, except that Bob rotates his polarizer by θ, while Alice returns hers to the vertical.

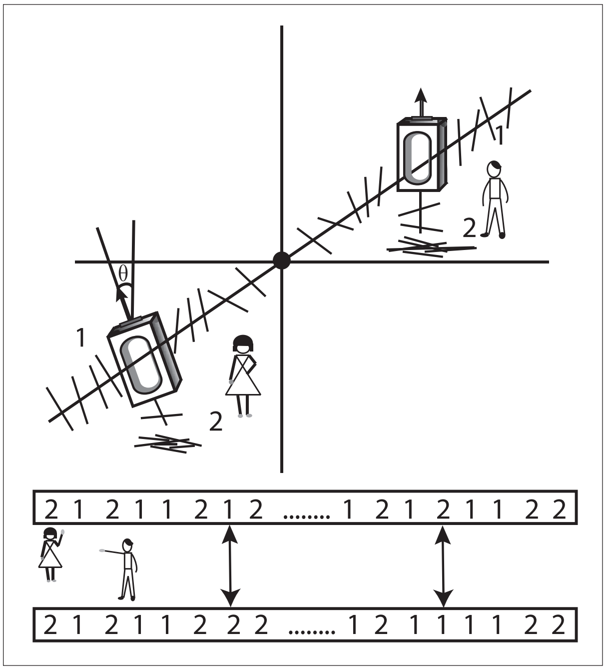

Fig. 4. Experiment II. Alice's polarizer rotated. Mismatches due to the rotation of Alice's polarizer.

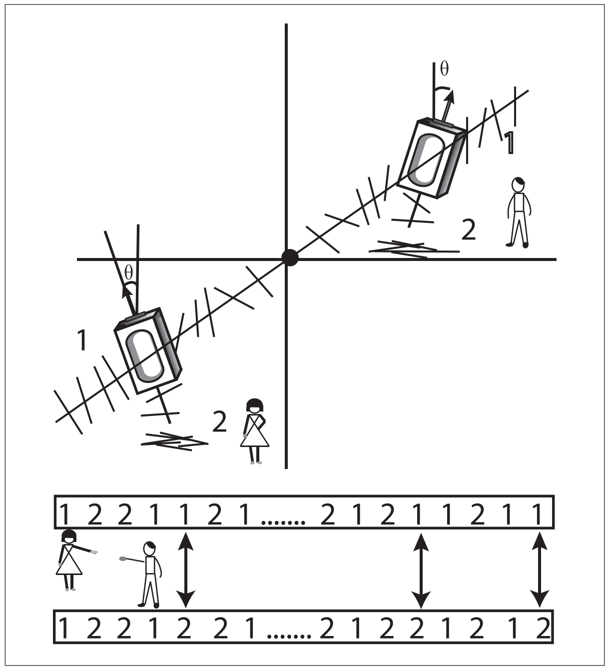

Fig. 5. Experiment IV. Both Alice's and Bob's polarizers rotated. Mismatches due to both rotations.

Since the situations are symmetrical, the mismatch rate again would be 5%, assuming that the number of "photon" pairs was large enough that statistical error was negligible.

Experiment IV:

This time Alice and Bob *both* rotate their polarizers by θ. If they rotated in the same direction, it would amount to no rotation at all; their polarizers would still be aligned. So they rotate their polarizers by θ in opposite directions.

Alice, rotating her polarizer by θ, changes the behavior of her "photons" by the same amount as in Experiment II. She changes what would have happened to 5% of her "photons." Similarly, Bob's polarizer rotation by θ changes the behavior of 5% of his "photons" from what would have happened.

Since Alice and Bob each changed the behavior of 5% of their "photons," and since every change could show up as a mismatch when their data streams are compared, we might expect a mismatch rate as high as 10%. There is no way to get a greater mismatch rate in a statistically large sample.

We might, however, get a *smaller* mismatch rate. Here's how: It might be that for some pairs of twin-state "photons," *both* Alice and Bob caused their twin to change its behavior. The "photons" of such twin-state pairs would thus behave identically. The data for such twin-state pairs would not be recorded as mismatches.

As an example of such a double change of behavior, consider almost vertical twin-state "photons" that would both go on Path 1 at Alice and Bob's polarizers if their axes were both vertical. If Alice and Bob each rotated their polarizers in opposite directions, as they did in Experiment IV, they could cause both these twins to go on their Path 2. They would thus not record this double change as a mismatch.

Because of such double changes, when Alice and Bob compare their data streams in Experiment IV, the mismatch rate will likely be *less* than the sum of the 5% mismatch rate Alice alone would cause *plus* the 5% mismatch rate that Bob alone would cause. Therefore, in Experiment IV the mismatch rate they will see is likely *less* than 10%. In a statistically large sample it *cannot* be greater.

That's it! We've derived a Bell inequality: *The mismatch rate when both polarizers are rotated by θ (in opposite directions) is equal to, or less than, twice the mismatch rate for the rotation by θ of a single polarizer.*

Since space is isotropic, a rotation of the two polarizers in opposite directions is equivalent to a rotation of only one by 2θ. Thus, the experimental results can be seen by, say, Alice rotating her polarizer by θ and then by $2\,\theta$.

Here's an intentionally ridiculous story emphasizing that the only *actual* assumptions in our derivation of a Bell inequality were reality and separability. Instead of talking of sticks and oval polarizers, we could have said that each "photon" is steered by a little "photon pilot" and that a polarizer is just a traffic sign indicating an "orientation" with an arrow. The "photon pilot" carries a travel document instructing him to steer his "photon" on Path 1 or Path 2 depending on the angle of the arrow. The hidden variable is now the physically real instruction printed on the pilot's travel document. His sister, piloting the "photon's" twin, follows her identical instructions at the polarizer she encounters with no regard for the behavior of her brother. This model yields the same Bell inequality.

Suppose *actual* experimental data violated Bell's inequality. That is, suppose that the actual mismatch rate for both rotations was *not* equal to, or less than, twice the mismatch rate for the rotation by θ of a single polarizer, but was *greater* than that. Since our Bell inequality was deduced assuming *only* reality and separability, its violation would mean that one or both of those assumptions *had* to be wrong. It would mean that our actual world lacks either reality or separability or both. We will see below that a violation in any one case (actual twin-state photons, for example) means a lack of reality or separability for everything such photons could possibly interact with. That is, in principle, *everything*.

The experimental tests

In 1965, when Bell's theorem was published, it was a mild heresy for a physicist to question quantum theory or even to doubt that the Copenhagen interpretation settled all philosophical issues. Nevertheless, as a physics graduate student at Columbia University in the late 1960s, John Clauser was intrigued.

Off to Berkeley as a post-doc to work on radio astronomy with Charles Townes, Clauser presented his idea for a test of Bell's inequality. Townes released him from his commitment to work on astronomy and even continued his financial support. With borrowed equipment, Clauser and a graduate student measured what we have called the "mismatch rate" for twin-state photons with polarizers set at different angles with respect to each other. They, in essence, did the Alice and Bob experiments we described.

They found Bell's inequality *violated*—in just the way quantum theory predicts.[10] To avoid a common misstatement, we emphasize that Bell's *inequality* was violated. Bell's *theorem*, the derivation of the inequality from the assumptions of reality and separability, is a mathematical proof not subject to experimental test.

What does quantum theory predict?

By how *much* the inequality was wrong is not crucial. The fact that the inequality was violated by *any* amount denies the reality or separability assumptions with which it was derived. (Were it *not* violated, quantum theory would have been shown wrong, but nothing would be proven about reality or separability. Incorrect assumptions can lead to some correct predictions.) Since the extent of the violation is not crucial, the following paragraph can be ignored in a less technical discussion.

A semi-classical calculation considering light's electric field gives the correct answer for the mismatch rate, even though it cannot deal with the photon correlations needed to establish the *meaning* of Bell's inequality. We note the following facts: 1) Alice's observation of a photon going through her, say, vertical polarizer means its twin at Bob's polarizer will be vertical. 2) The fraction of light intensity (or photons) *not* going through Bob's polarizer—the mismatch rate—is proportional to the square of the component of electric field *perpendicular* to Bob's polarizer axis. 3) This is proportional to the square of the sine of the angle θ of Bob's polarizer to the vertical.[11] Thus, the actually observed mismatch rate—and that given by quantum theory—is proportional to $\sin^2\theta$. 4) Bell's inequality thus states: $\sin^2(2\theta) \leq 2\sin^2\theta$. Try this for $\theta = 22.5°$, $2\theta = 45°$. We get $0.5 \leq 0.29$, obviously not true. We thus see that in the actual world, Bell's inequality can be strongly violated.

The bottom line for the experimental results

Clauser's experiments ruled out what is sometime called "local reality." The experiments showed that the properties of objects in our world have an observation-created reality *or* that there exists a connectedness beyond that mitigated by ordinary physical forces, or both. In these experiments, quantum theory survived its most serious challenge in decades.

Clauser writes: "My own . . . vain hopes of overthrowing quantum mechanics were shattered by the data." Confirming quantum theory's predicted violation of Bell's inequality, he showed instead that a description of our world with both reality and separability would never be possible. Before Clauser's result, we could not know this.

Unfortunately for Clauser, in the 1970s investigation of the fundamentals of quantum mechanics was not yet considered proper physics in most places. (Still so in some places.) When he sought an academic position, "What has he done besides checking quantum theory? We all *know* it's right!" was a typical misunderstanding of Clauser's accomplishment. Clauser got a job in physics, but not one in which he could participate in the wide-ranging investigations he launched.

A decade later, with more advanced technology and a somewhat more receptive atmosphere for exploring quantum fundamentals, Alain Aspect in Paris duplicated Clauser's results with far greater accuracy, showing that the violation of Bell's inequality was by just the amount predicted by quantum theory to extremely high precision.[12] His faster electronics also established that *no* physical effect could propagate from one polarizer to another in time for the observation of one photon to affect the other. This closed a small loophole in the experiments done by Clauser, whose electronics were not fast enough to establish this. If John Bell had not died, Bell, Clauser, and Aspect might well share a Nobel Prize.

Where does a violation of Bell's inequality leave us?

Experiments demonstrate that our world cannot have *both* reality and separability. Quantum theory says it has neither. Where does this leave us?

"Reality" has been our shorthand for physically real properties existing locally, even within unobserved objects. We must be careful about "unobserved." Any interaction with the environment can be, for all practical purposes, an observation. And since it is practically impossible to isolate a macroscopic object, large objects are essentially real, for all *practical* purposes. The wave function of any microscopic object is said to "collapse" on interacting with (on being "observed" by) a macroscopic object. The microscopic object thereby randomly acquires a particular one of the potentialities allowed by the wave function.

"Collapse" is, however, a process unexplained within quantum theory. Today, instead of collapse, one might refer to "decoherence," the much-studied process whereby a microscopic system's coming into contact with the macroscopic environment causes the phases of different parts of its wave function to decohere with extreme rapidity. Decoherence assures us that we can ignore the reality problem for the macroscopic objects we deal with directly. Decoherence resolves the reality paradox, which is often called the "quantum measurement problem," for all practical purposes. Of course there never was a problem for all *practical* purposes.

But quantum theory is seamless, with no boundary between the microscopic and the macroscopic (it is even applied to simple models for black holes and the big bang). In principle, if the polarization of a yet-unobserved photon is not a reality, neither is the fired or unfired state of a Geiger counter if it could be isolated from the environment and set to fire only if the photon is vertical. Though such non-reality might have little *practical* consequence, it is something to ponder.

"Separability" has been our shorthand term for objects being affected only by physical forces traveling no faster than the speed of light. Without separability what happens at one place can instantaneously influence what happens far away without any physical force connecting the two events. Experiments have demonstrated such influences extending beyond 100 km. Quantum theory has this connectedness, or entanglement, extending over the entire universe. And if the Geiger counters observing twin-state photons could both be isolated from the environment, they would be entangled with each other. In principle any two objects that have ever interacted are forever entangled. The behavior of one instantaneously "influences" the other—and the behavior of everything entangled with either. We talk in terms of twin-state photons only because that situation is readily described and subject to experimental test. For practical reasons, however, quantum entanglement can be displayed only for very small objects.

The Bell's theorem experiments establish only that our actual world does not have *both* reality and separability. But accepting the EPR definition of reality, and the experimental results for twin-state photons (our Experiment I), which demonstrate such a reality, the lack of separability in itself is established.[13]

A lack of separability should not be confused with a simple classical correlation. It is not the same as, for example, identical twins having the same eye color. A lack of separability means that what happened at Alice's polarizer instantaneously influences what happens at Bob's. Decoherence assures us that for macroscopic objects, this lack of separability, this quantum entanglement, is generally too intricate to discern. But that is not *completely* so. Quantum entanglements of the essentially macroscopic structures of future quantum computers are expected to make use of quantum connectedness, and some have already been demonstrated.[14]

Interpretations of quantum theory explore what might actually "be going on" in the world. Several interpretations currently compete, but all deny separability. The most famous competitor of the Copenhagen interpretation is the bizarre "many worlds" interpretation. It accepts the many possibilities given by the wave function as simultaneously existing actualities, with the observer (or the observer's mind) present in each of them. Another currently discussed interpretation, the Bohm interpretation, postulates real particles in *addition* to wave functions, but then requires a "quantum potential" instantaneously connecting all things that have ever interacted. Some physicists, rejecting the need for *any* interpretation, contend that a for-all-practices-purposes theory is all that physics need provide.

Seeking a more intuitively reasonable description of the world, one might go beyond interpretations to look for a breakdown of quantum theory for macroscopic systems. To date, experiments seeking to challenge quantum theory in this way have only displayed the theory's predicted results. Even were quantum theory not to apply for large objects, we would still be left with the problems of reality and separability for the small objects that large objects are made of.

Actually, there is one way to avoid "nonphysical" faster-than-light influences. There was an assumption in our Alice and Bob story (and also in Bell's rigorous treatment) that we have not mentioned: We implicitly assumed that Alice and Bob are what Bell called "freely operating experimenters." We could assume instead that the world is totally (*totally*!) deterministic. It would be a world in which experimenters could not freely choose their polarizer angles. We would then not need Bohr's faster-than-light influences.[15]

Bell considered such a possibility and concluded: "[T]his way of arranging quantum mechanical correlations would be even more mind boggling than one in which causal chains go faster than light. Apparently separate parts of the world would be deeply and conspiratorially entangled, and our apparent free will would be entangled with them."[15]

The enigma physics has encountered can intrigue students at any level. An instructor might well take advantage of this.

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