Chapter 9

Relativity I

The postulate and the speed limit

Alice laughed: "There's no use trying," she said; "one cannot believe impossible things."
"I daresay you haven't had much practice," said the Queen. "When I was your age, I always did it for half-an-hour a day. Why sometimes I've believed in as many as six impossible things before breakfast."

Lewis Carroll, "Through the Looking Glass"

One reason for interposing two chapters on Relativity between classical mechanics and quantum mechanics is to practice believing "impossible" things. Coming to understand Einstein's Theory of Special Relativity is humbling. Things you knew for sure turn out to be wrong. Grasping these ideas must expand the mind, there would be no room for them otherwise.

Galileo taught us to frame careful questions for Nature and adjust our intuitions to her answers, however strange they seem to us. Relativity powerfully reinforces that injunction.

A wonderful thing about Special Relativity is that it can be understood with a minimal physics background and some quite elementary mathematics. Possibly the greatest impediment to understanding is the difficulty of *believing* what you learn.

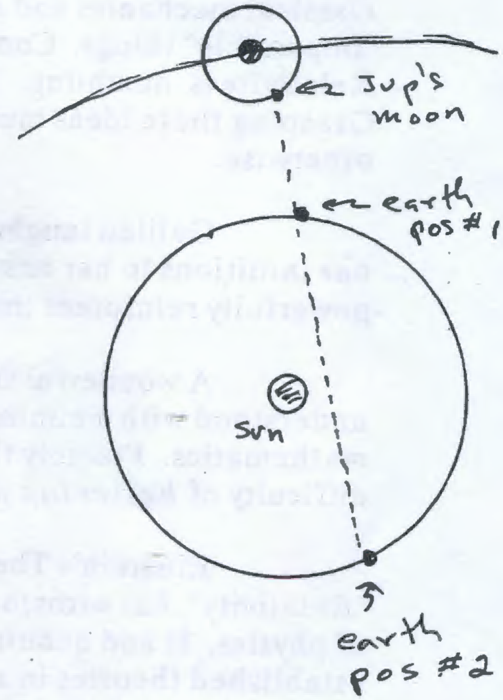
Einstein's Theory of Special Relativity, which is usually just called "Relativity", has withstood extensive tests. It is today the logical basis of much of physics. It and quantum mechanics are without doubt the most well-established theories in all of science. Today, one who denies the basic validity of Special Relativity risks being considered a crackpot.

The theory is "special" in the sense that it does not address questions of acceleration and gravity. It is formulated for observers who are not accelerating and not in changing gravitational fields. Newton's Second Law has the same restriction to non-accelerating reference frames, or to "inertial" systems. A different theory, Einstein's Theory of General Relativity is in fact a theory of gravity. It is "general" in the sense that it includes Special Relativity as one particular, or "special", case. While the seventy-year-old General Theory of Relativity is still the leading contender among theories of gravity, it is less well-established than the Special theory, and there is still active examination of its validity. For us "Relativity" will mean the Special Theory.

294

The propagation of light

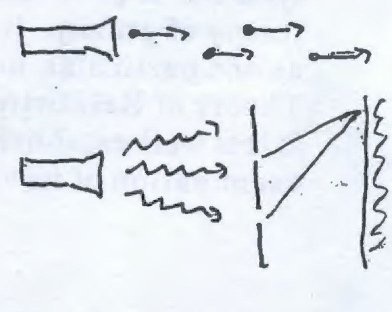
The speed of light: When you turn on the light, do not the far ends of the room become bright immediately? It might seem that light spreads out from its source to the illuminated objects instantaneously. Galileo was one of the first to suggest that it took light some small time to travel. If only he could achieve sufficient skill at uncovering lanterns rapidly, Galileo felt he would be able to measure its speed of transmission. He was basically right. The distances he worked with, however, were too small, and the light covered them too quickly.



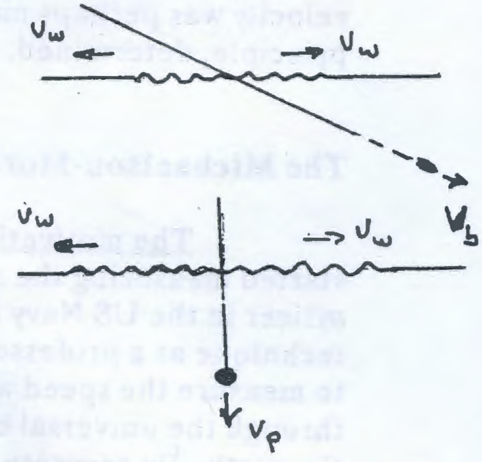
A few decades after Galileo, the Danish astronomer Ole Roemer recorded the exact times a moon of Jupiter was eclipsed behind the planet. He noticed that these eclipses were not completely regular. They came later than exact regularity would predict when the earth was on the far side of its orbit from Jupiter. He correctly decided this was because it then took the light from the eclipsing moons longer to reach earth. From this time discrepancy, he calculated the speed of light.

By the middle of the 19th Century, good measurements of the speed were being made between mountain tops. Light travelled at about 3×10^8 m/s. Maxwell's great triumph was being able to calculate this speed from measurements on stationary electric charges and magnetic fields. He convinced his colleagues that light waves were in fact waves of electric and magnetic fields.

The "ether": If light were particles shot out of glowing bodies, a medium for its transmission would not be necessary. Particles could move through empty space. But interference proved light was a wave. Didn't something have to be waving?



The speed of a wave does not depend on the speed of the object launching it. The ripples caused by the speeding bullet slanting into the water go no faster than those from the dropped pebble. The properties of the medium in which a wave propagates determine how fast it goes. From those properties Maxwell actually calculated the speed of light.



It seemed a strange medium which carried light. It must fill all space, since light comes to us from distant stars. It must readily stream through solid bodies--at least those which transmit light. It must be extremely tenuous since we feel no resistance in moving though it. We see nothing of this ethereal, all-pervasive substance except that it carries light. "Ether" was a well-chosen name for it[1]

With what speed does the earth move through the "ether"? The question *itself* was more fundamental than any answer giving some particular speed. Until now, only the *relative* motion of objects was meaningful. But now, *because--and only because--light was a wave moving in an all-pervading universal medium, things are different*! One could determine *absolute* velocities, velocities relative to this universal medium. The "ether" was a "cosmic hitching post" defining absolute rest.

According to Galilean relativity and Newton's mechanics, only relative velocity was meaningful. No mechanical experiment could establish absolute rest. But now it seemed, with measurements on light, there was no longer a complete equivalence of inertial frames. An absolute

1 I will keep the word "ether" in quotes. We will soon see it need not exist. The concept is, however, of historical interest, and thinking in terms of it allows us to understand the motivation of the Michaelson-Morely experiment we soon talk about. The "ether" concept is not intrinsic to the problem encountered. When we abandon "ether", the problem stays. The word "ether" could be replaced by "vacuum".

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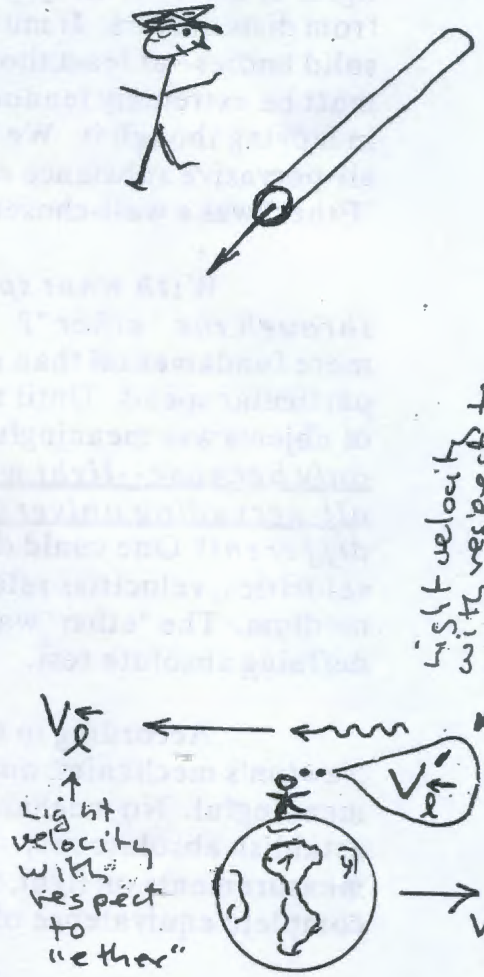
velocity was perhaps meaningful and could be, in principle, determined.

The Michaelson-Morely experiment

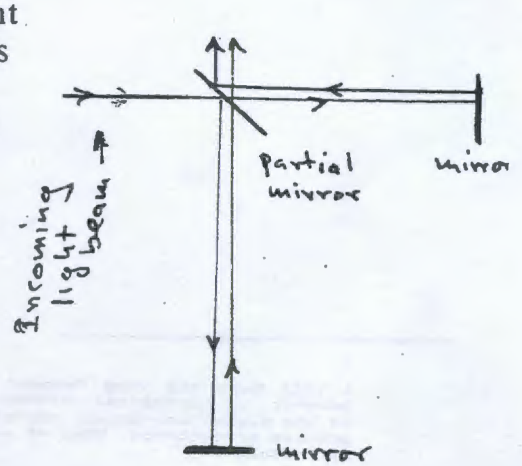


The motivation: Young Albert Michaelson started measuring the speed of light as a scientist-officer in the US Navy in 1878 and then refined his technique as a professor at Case Institute. He wanted to measure the speed with which the earth moved through the universal ether--the absolute velocity of the earth. By accurate measurements of the speed of light through the "ether" in different directions he would calculate the speed of the earth.

The method: Michaelson's method[2], was not quite the one we discuss and diagram, but the principle is the same and the result equivalent. Our method provides a simpler explanation, though it would be a more difficult experiment to actually do, especially in Michaelson's day.

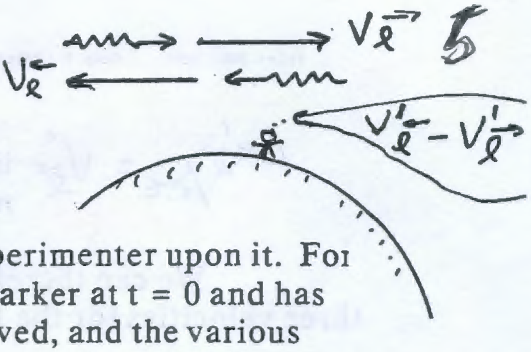
In Chapter 3, we derived Galileo's rule for the addition of velocities with the example of a bird flying past a moving wagon. Let us essentially repeat that derivation, but with the moving wagon replaced by the moving earth in space, and the flying bird by a short flash of light moving past.

We will do this for two cases: the first for the flash, or light "pulse" moving from west to east, the direction of the earth's motion around the sun, and the second with the light going in the opposite direction. You would expect a slower speed for light moving through space along with you than when it is moving against you. In each case, an observer on earth, could measure the speed of the passing light pulse. From the difference in these two measurements, he should be able to calculate the



2 He used a technique wherein light from a laboratory source travelled along two paths at right angles and was then brought together by an arrangement of mirrors to cause an interference pattern. By looking for changes in the pattern as he rotated the device, now called a "Michaelson interferometer", he could detect any difference in the wavelength for light moving in two directions and thus, by v = f \, in the velocity relative to the ether in the two directions.

the "Michaelson



speed of the earth through space, or through the "ether".

Let us consider the moving earth with our experimenter upon it. For our first case, a light pulse passes our experimenter's marker at t = 0 and has moved some distance by t = Δt. The earth has also moved, and the various distances are indicated in Figure 8.1. The little arrows over the subscripts indicate the direction of the light. The primed (') values are those measured by an experimenter on the moving earth. The unprimed values would be those measured by someone at rest in space. i.e., at rest in that special system in which light moves at the same speed in all directions, at rest in the "ether".

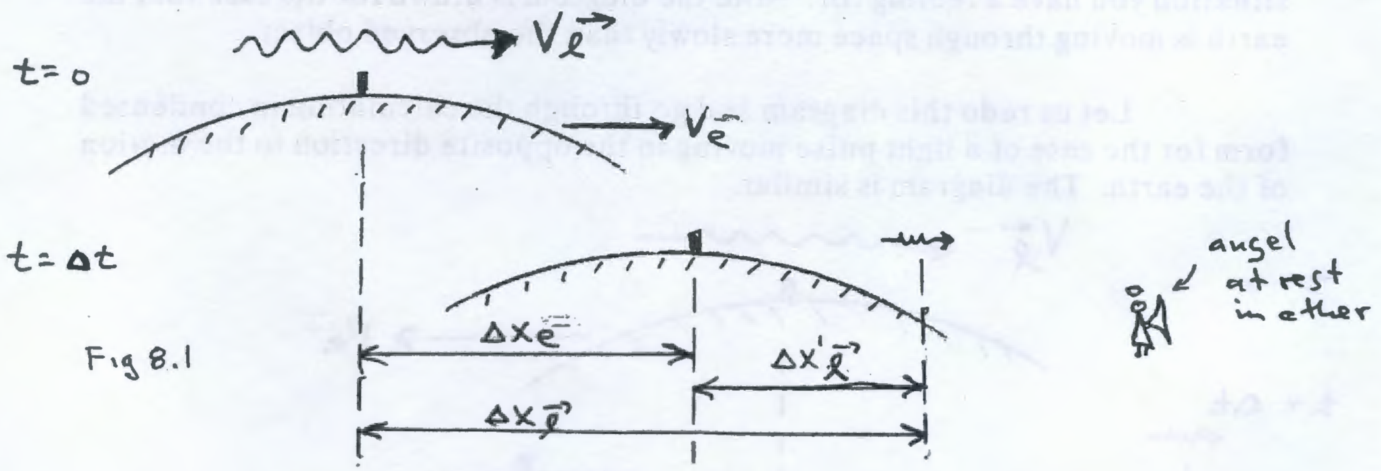


Fig 8.1

The distances indicated in the diagram are the distance the earth has moved, the distance the light has moved in the frame of the earth, and the distance the light has moved in the frame of the person at rest in space. All these motions took place in the time Δt.

The relationship between these distances, by inspection of the diagram is

$$\Delta x_{\vec{\ell}} = \Delta x'_{\vec{\ell}} + \Delta x_e \qquad 9.1$$

We can divide Equation 9.1 on both sides by Δt to get

$$\Delta x_{\vec{\ell}}/\Delta t = \Delta x'_{\vec{\ell}}/\Delta t + \Delta x_e/\Delta t \qquad 9.2$$

The terms in Equation 9.2 have the obvious meanings

$\Delta x_e/\Delta t = V_e$ is the velocity of the earth through space.

$\Delta x_{\vec{\ell}}/\Delta t = V_{\vec{\ell}}$ is the velocity of the light pulse moving to the right--as measured by the person at rest in space.

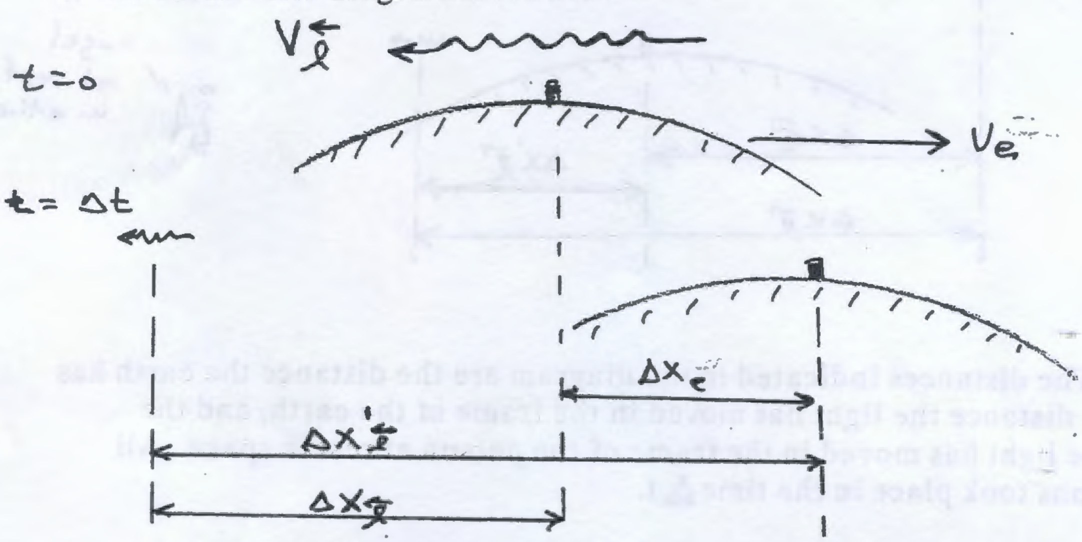
$$\Delta x'_{\ell\rightarrow}/\Delta t = V'_{\ell\rightarrow}$$ is the velocity of the light pulse moving to the right--as measured by the experimenter on the earth.

We can therefore rewrite Equation 9.2 as a relationship between the three velocities for the light travelling in *same* direction as the earth.

$$V_{\ell\rightarrow} = V'_{\ell\rightarrow} + V_e \qquad 9.3$$

This equation is nothing but Galileo's intuitively obvious relation written for a special case. You might wish to try some numbers in it appropriate to a situation you have a feeling for. Note the diagram is drawn for the case that the earth is moving through space more slowly than the observed object.

Let us redo this diagram and go through the calculation in condensed form for the case of a light pulse moving in the opposite direction to the motion of the earth. The diagram is similar.



The distances marked can now be seen to have the relationship

$$\Delta x_{\ell\leftarrow} = \Delta x'_{\ell\leftarrow} - \Delta x_e \qquad 9.4$$

And again, we can divide through by Δt, identify the various velocities, and write for the light travelling in the direction *opposite* to the earth

$$V_{\ell\leftarrow} = V'_{\ell\leftarrow} - V_e \qquad 9.5$$

But now, the right hand side of Equations 9.3 and 9.5 are the speeds of light traveling in opposite directions *as measured by someone at rest in*

198

the frame in which the speeds in both directions are the same. Therefore[3]

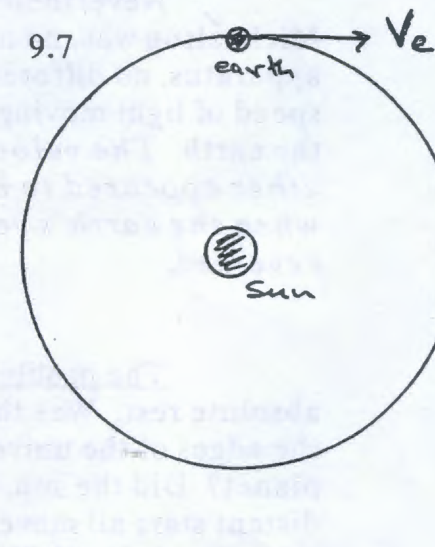
$$V_{\ell\rightarrow} = V_{\ell\leftarrow} \qquad 9.5a$$

and we can equate the right sides of Equations 9.3 and 9.5 to get

$$V'_{\ell\rightarrow} + V_e = V'_{\ell\leftarrow} - V_e \qquad 9.6$$

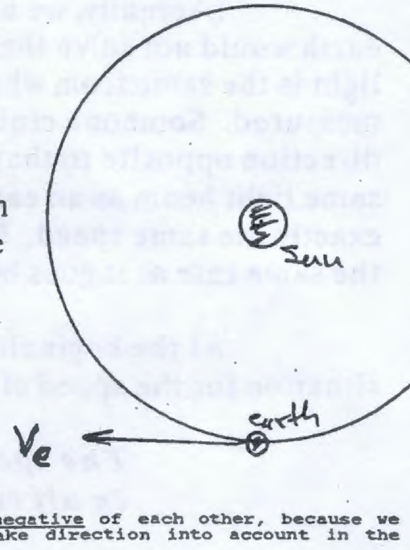
We can solve Equation 9.6 for the speed of the earth through space. This should be the *absolute velocity of the earth*, its velocity with respect to the frame of reference in which the speed of light is the same in all directions, or the earth's velocity with respect to the "ether".

$$V_e = (V'_{\ell\leftarrow} - V'_{\ell\rightarrow})/2 \qquad 9.7$$



These two speeds of light when it was travelling with and against the direction of motion of the earth could be measured. Considerable accuracy is required because the speed of the earth in its orbit about the sun is ten thousand times less than the speed of light. To expect to see a difference for the two directions, one would need to measure light speeds (or, at least the difference between two of them) to better than one part in ten thousand of the speed of light. Not an easy task, but Michaelson an Morely were up to it.

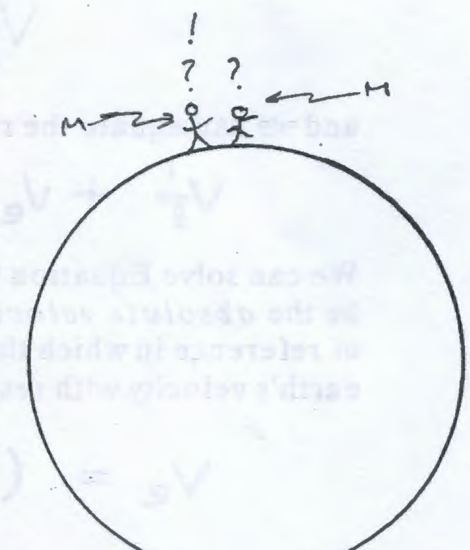
The puzzling result was that no difference could be detected between $v'_{\ell\rightarrow}$ and $v'_{\ell\leftarrow}$. The speed with which light went by in the direction of the earth's motion was no different than the speed with which it went by in the opposite direction. The speed of the earth through space was zero! $v_e = 0$.



This could perhaps be explained if the earth just accidentally *happened* to have a velocity so close to zero with respect to the ether that no motion could be detected. The experiment may not have been accurate enough. Perhaps the absolute speed of the earth through space were less than one part in ten

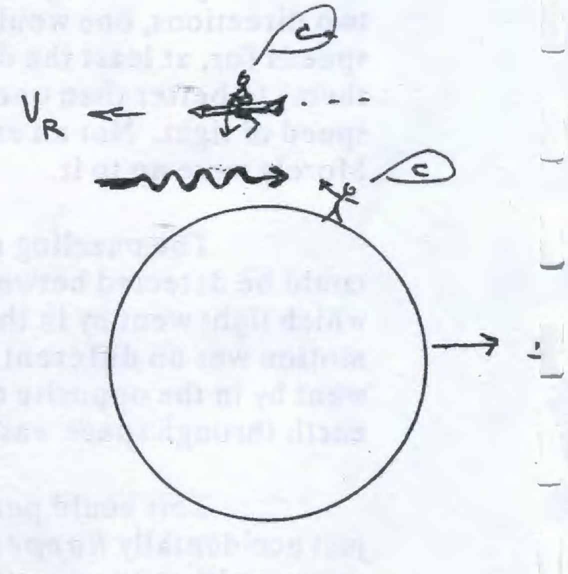
[3] The two velocities are equal to each other, not equal to the negative of each other, because we chose to call our velocities and distances all positive numbers and take direction into account in the derivation of the equation.

thousand of the speed of light. That seemed unlikely, but a possibility.



However, this could be checked. Six months later the earth would be on the other side of its orbit about the sun. Its orbital velocity would be reversed. Its direction through the "ether" would not necessarily be exactly reversed, because the sun and the entire solar system might be moving. But in six months, the earth's velocity would change by twice its orbital velocity of 3×10^4 m/s. One could repeat the experiment then.

Nevertheless, no matter how careful Michaelson was, no matter how much he improved his apparatus, no difference could ever be detected in the speed of light moving with or against the motion of the earth. *The velocity of the earth through the ether appeared to be zero--and stayed zero even when the earth's velocity about the sun reversed.*



<u>The problem:</u> The earth seemed to be at absolute rest. Was the "ether", which had to extend to the edges of the universe, fixed with respect to our planet? Did the sun, the other planets, and the distant stars all move with respect to that fixed "ether" and a fixed earth? Was Aristotle right after all? What was a *reasonable* explanation for this?

(Actually, we now know even a stationary earth would not solve the problem. They speed of light is the same from whatever platform it is measured. Someone cruising by on a fast rocket in the direction opposite to that of earth and looking at the same light beam as an earth observer would measure exactly the same speed. It would pass the rocketeer at the same rate as it goes by the earthling.)

At the beginning of the 20th Century, the confounding experimental situation for the speed of light in vacuum could be summarized:

The speed of light is measured to be c in all reference frames.

Many tried to explain this strange result, but without success.

The Gordian knot: Gordius, King of Phrygia, tied two ropes together with a knot so complex, the ropes seemed impossible to separate. Reliable mystics divined: *he who separated the ropes would rule the world.* Many pulled at the twisted coils in vain. One day a young fellow came to the knot, but didn't try to undo it. He separated the ropes with a swing of his sword. Alexander the Great went on to conquer the world.

The postulate of Special Relativity: In 1905, Albert Einstein, still a clerk in the Swiss Patent Office, published an answer to the problem of the invariant speed of light. Whether or not his answer satisfies one immediately as an "explanation" can be argued, but it is surely the solution.

Einstein argued that we have discovered, by these experiments, a new fundamental law of nature. We should state it clearly as a postulate, deduce its consequences, and test them experimentally.

The law nature displays to us is:

The speed of light <u>is</u> the same in all reference systems.

If that's true, forget about the "ether"! There is now no need, or even a role, for it. The "ether" existed only to define *that particular reference system* in which the speed of light is the same in all directions, the system in which the medium carrying the light waves is at rest. If the speed of light is the same in *all* reference systems, *any* reference systems can be considered to be at rest.

No measurement of any *absolute* velocity is possible. Only relative velocities are meaningful, hence Einstein's Theory of "*Relativity*". All constant velocity systems are equivalent. Therefore, another way to phrase Einstein's postulate: *The laws of physics are the same in all inertial systems. Any non-accelerating system can be considered at rest.*

When the postulate of Relativity is stated this way, we can see that actually doesn't depend on any property of light. It came about historically by measurements on light, and in fact the speed at which light (and some other things) travel is germane. But we will see it applies much more generally and, in principle, *could have been* deduced without reference to light at all.

Stated as an equivalence of all inertial systems, the postulate of Relativity sounds almost innocuous. Doesn't this just bring us back to Galileo's idea of relative motion? No. If we accept Einstein's postulate, it takes care of the puzzling Michaelson-Morely experimental results in a formal sense--it does so by fiat. But it does violence to our intuitive understanding of time and space. It was this intuitive understanding which gave us Equations 9.1, 9.2. On it we based our entire discussion of the speed of something seen from different moving systems here and in Chapter 3. How can those equations and that most reasonable discussion with which they were derived be wrong? There is indeed a fundamental error.

We must understand that error. But for now let's deduce some consequences predicted by Einstein's postulate and see if they are in fact confirmed by experiment. They are, even if they are almost impossible to believe. By passing our "chip and hit tests" the postulate compels consensus. We will find ourselves forced to accept a new, and almost ridiculous, concept of the space and time we occupy.

Testing the postulate of Relativity

For some scientific postulates, experiments which would tend to confirm them, or might decisively refute them, are immediately evident. Galileo's postulate that heavy and lighter objects fell at the same rate, for example, readily suggests the experiment of measuring speeds of fall. The test of Newton's Second Law by measuring forces and accelerations and substituting these numbers into F = Ma seems straightforward.

What are the testable experimental predictions of the speed of light being the same in all reference frames? Well, you could measure the speed of light and see that it was the same no matter how fast you moved. But that's not a fair test. The postulate was *created* to account for that observation.

<u>Our program to examine the postulates:</u> We proceed by a technique developed by Einstein himself. He made up little stories, or "gedanken" (thought) experiments--the German word is often used--to illustrate the new nature of time and space. Once that is clarified by the stories, actual testable consequences of the postulates present themselves.

In these stories, we will look at the same events from two different reference systems moving with different velocities. An observer in either can, by the Relativity postulate, legitimately consider himself at rest, and the other as moving. We will analyze the what these fictional observers report. From that we will deduce experimental predictions of Einstein's postulate. We then

202
15ee

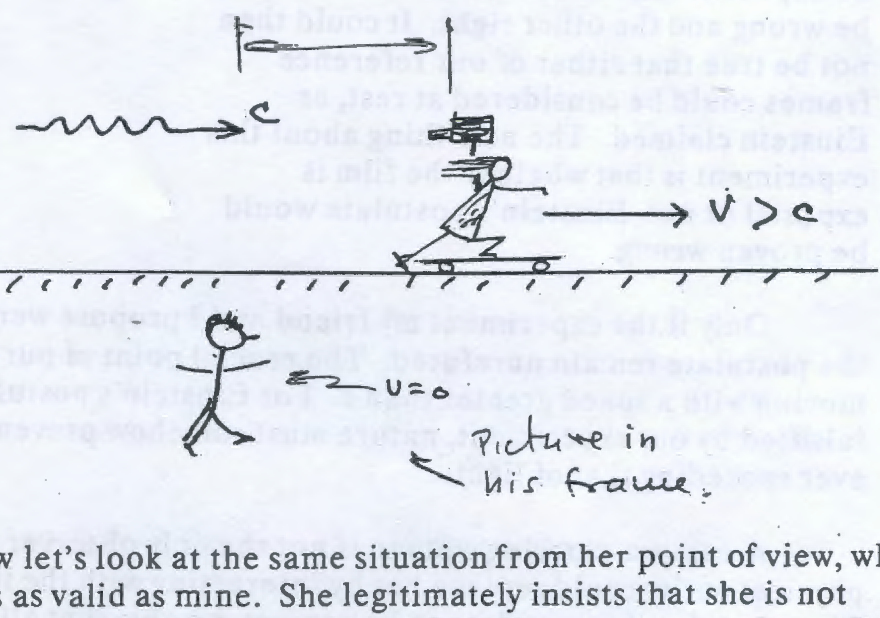
see whether actual experiments accord with or refute these predictions and the postulate.

Let us tell our first story and deduce its consequence.

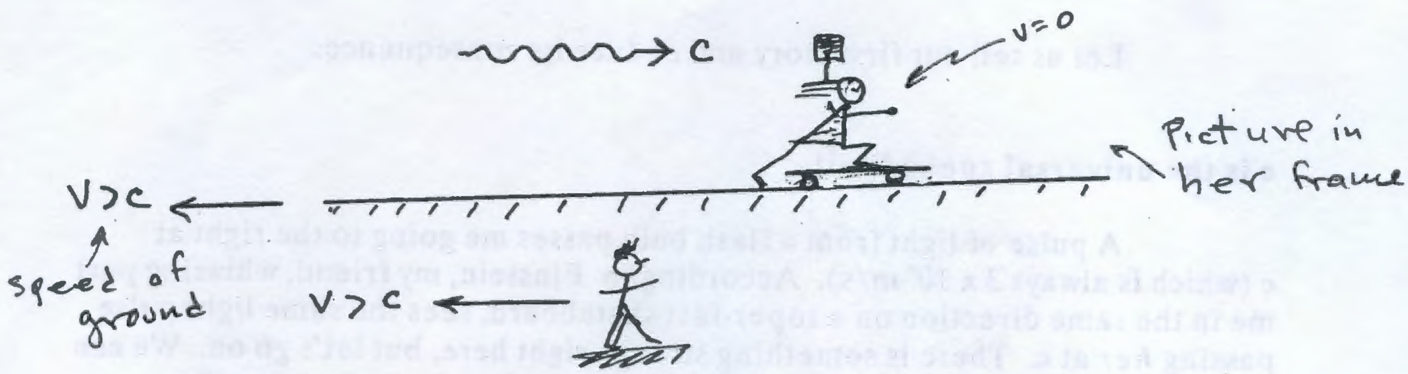
c is the universal speed limit.

A pulse of light from a flash bulb passes me going to the right at c (which is always 3×10^8 m/s). According to Einstein, my friend, whizzing past me in the same direction on a super-fast skateboard, sees the same light pulse passing *her* at c. There is something strange right here, but let's go on. We can each learn of the light speed the other reports, and perhaps question the other's measurement technique, but, if Einstein's postulate is correct, there is no physical event that could demonstrate one of us right and the other wrong.

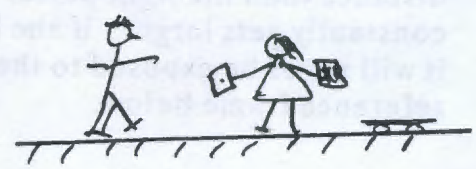
Can this be so? Let's try to refute the postulate. She and I arrange for her to scoot past me at a speed *greater* than c. And, just at the moment the light pulse goes by me, she will plan to be a few feet ahead of it. Her going faster than the light means that in each time interval she will cover a greater distance than the light pulse. The distance between her and the pulse constantly gets larger. If she has a camera pointed back at the light, the film in it will never be exposed to the light pulse. I illustrate this story, from *my* reference frame below.



But now let's look at the same situation from her point of view, which Einstein claims is as valid as mine. She legitimately insists that she is not moving at all. She is just pushing on the skateboard to stay in one place. She must do that because the road and are moving to the left with a velocity greater than c. The light pulse will approach her at c and simply enter her camera. My motion is irrelevant to her photography. The story from *her* frame of reference is illustrated below.



My friend and I will have a point of disagreement which is physically resolvable. She will claim that her film is exposed to the light pulse, and I will say that cannot possibly be. To see who is right, after she closes her shutter, she can stop her skateboard, and together we can examine the film. It will either exposed or it will not be exposed. One of our points of view must be wrong and the other right. It could then not be true that either of our reference frames could be considered at rest, as Einstein claimed. The nice thing about this experiment is that whether the film is exposed or not, Einstein's postulate would be proven wrong.



Only if the experiment my friend and I propose were impossible could the postulate remain unrefuted. The crucial point of our proposal was her moving with a speed greater than c. For Einstein's postulate not to be falsified by our experiment, nature must somehow prevent her speed from ever exceeding that of light.

A camera-carrying woman is not the only observer possible, any physical entity could replace her by interacting with the light pulse. Therefore for the postulate to be correct, no object at all could ever move faster than light. c would then be the natural speed limit of the universe. Is this actually true?

<u>Experimental test:</u> The experimental test is therefore to try to get something to move faster than the speed of light. To the extent that *every* sophisticated attempt to do that consistently fails, we would rely more and more on the correctness of Einstein's postulate. But if we can ever get anything to move faster than light, we force the abandonment of Einstein's postulate of Relativity.

Let's list the speed of some of the fastest items around, other than light itself.

Fastest cars (almost 500 mph)--- 0.0000008c

Fastest plane (8,000 mph)------- 0.00001c

Fast rocket (80,000 mph)-------- 0.0001c

Electrons in TV tube------------ 0.01c

Electrons in x-ray tube--------- 0.1c

Helium nuclei in Bevatron----------- 0.8c

Getting close!

But look what now happens:

Beta-ray electrons-------------- 0.99c

Betatron electrons-------------- 0.9999c

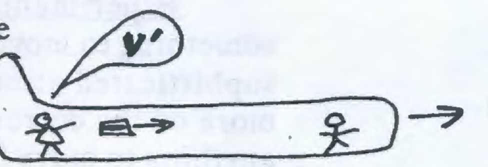
Electrons at SLAC[4]-----------------0.99999998c

Fast cosmic ray particles------0.999999.?.999c
(from outer space)

From this data it seems we can get things extremely close to the speed of light, but exceeding it somehow appears impossible. $c = 3 \times 10^8$ m/s seems indeed to be a universal speed limit. Before Einstein came along no one ever suspected a natural speed limit. Score one (at least) for the Relativity postulate.

[4] Electrons at SLAC are sped up at the <u>S</u>tanford <u>L</u>inear <u>A</u>ccelerator <u>C</u>enter in a 2-mile long vacuum tube. The object is to keep pushing them to get them to go faster and faster to crash into atomic nuclei to study what they and their parts are made of.

<u>The relative velocity equation:</u> If it's true that nothing can go faster than light, what's wrong with the relative velocity equation we derived in Chapter 4? That equation is, with slightly different notation,



$$v = v' + u, \qquad 9.8$$

where v is the velocity of an object as seen from the frame we consider at rest, v' is its velocity in the moving frame, and u the velocity of the moving frame.

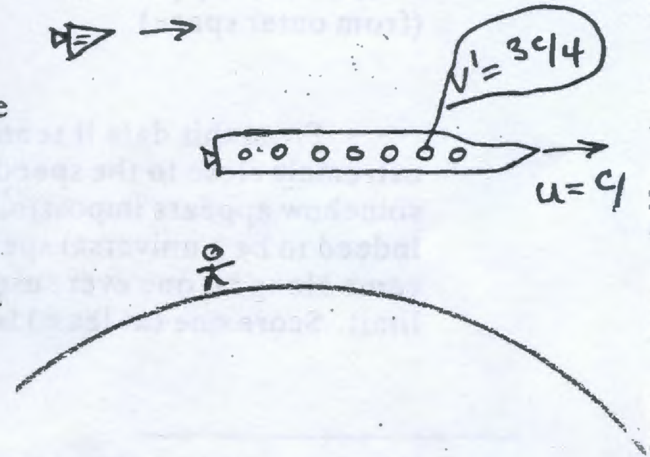
Doesn't this say that if the skateboarder moves with a velocity u, even somewhat less than c, and she sees the light going past her at v' = c, I, in the rest frame must see the light beam go by at a speed v greater than c. Yes, it says that, but the equation is wrong.

Using the postulates of Relativity, we can derive the correct formula for the relative velocities. Instead of Equation 9.8, the relativistically correct formula is

$$v = (v' + u)\Big/\left(1 + uv'/c^2\right) \qquad 9.9$$

Now, at velocities small compared to the speed of light, the velocities we are used to, the term uv/c^2 in Equation 9.9 is extremely small compared to 1. The denominator of Equation 9.9 is then almost unity, and the relativistic equation is essentially the same as Equation 9.8. For velocities small compared to that of light our old ideas work well.

But try Equation 9.9 for some velocities closer to c. Consider two spaceships of an advanced society going past each other in our sky. The first, a large freighter, passes the earth at only half the speed of light, u = c/2. The other, a faster patrol ship, is overtaking it. We overhear the navigation officer on the freighter report that the patrol is overtaking him at a rate of v' = 3c/4. By our *old* considerations, we might conclude that the patrol is going by <u>us</u> at



$$v = c/2 + 3c/4 = 5c/4 = 1.2\ c.$$

This would mean we would see something exceeding the speed of light, a violation of the Relativity postulates (and of all experimental evidence).

Using the correct relative velocity relation, Equation 9.9, we get

$$v = (c/2 + 3c/4)/[1 + (c/2)(3c/4)/c^2] = 10c/11 = 0.9\,c,$$

which is close to c, but does not exceed it. Try some other speeds in the equation. Try v' = c, and u = anything; or u and v both equal to c. It's an interesting equation to play with.

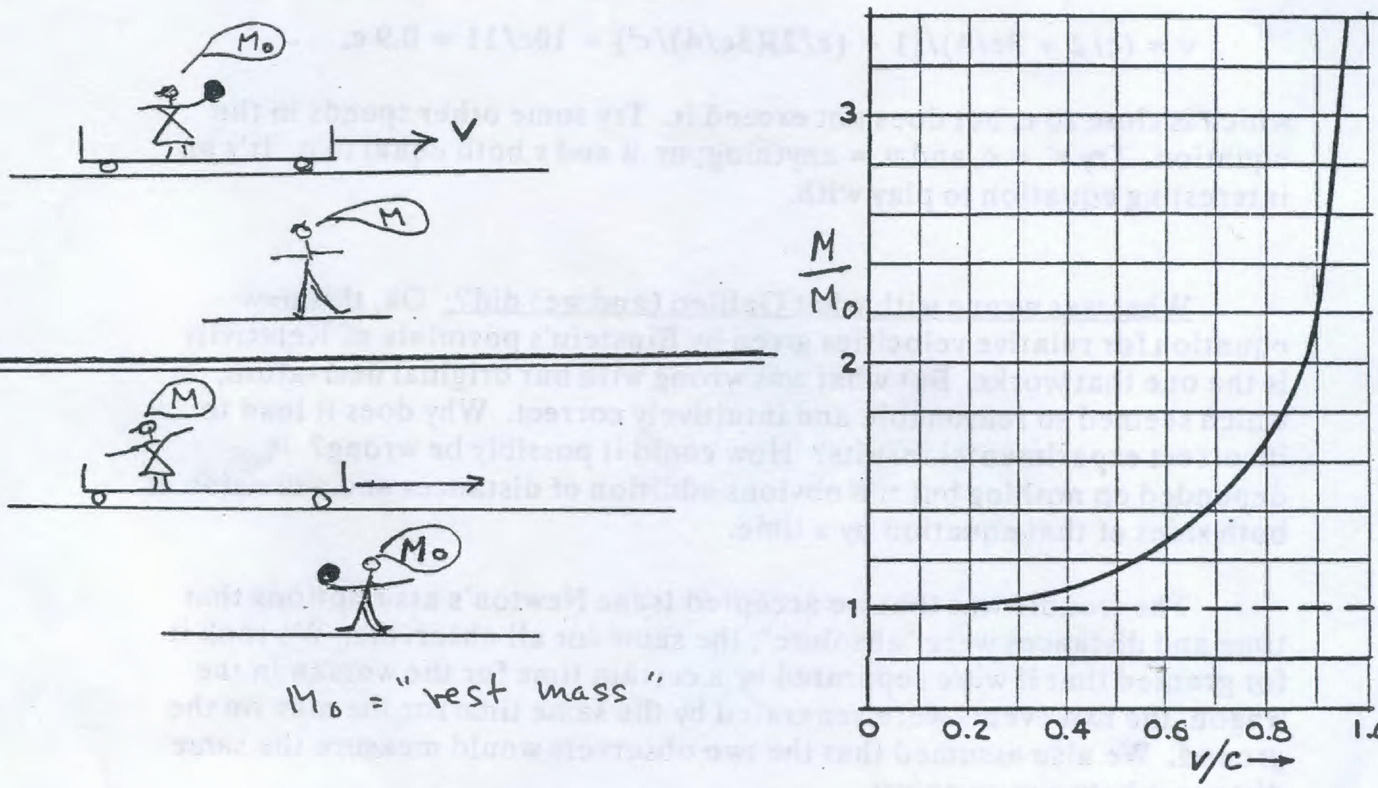
What was wrong with what Galileo (and we) did?: Ok, this new equation for relative velocities given by Einstein's postulate of Relativity is the one that works. But what was wrong with our original derivation, which seemed so reasonable and intuitively correct. Why does it lead to incorrect experimental results? How could it possibly be wrong? It depended on nothing but the obvious addition of distances and a division of both sides of that equation by a time.

The trouble was that we accepted Isaac Newton's assumptions that time and distances were "absolute", the same for all observers. We took it for granted that if were separated by a certain time for the woman in the wagon, the two events were separated by the same time for the man on the ground. We also assumed that the two observers would measure the same distances between to points.

These assumptions are incorrect when velocities are comparable to that of light. In the next chapter, we see that we must regard the passage of time and the extent of a length as *relative* things, things which depend on the reference system relative to which they are measured. (Hence the name, "Theory of Relativity".)

How is the universal speed limit enforced? If we keep pushing something, doesn't it keep accelerating? Therefore if we *keep* pushing it can't we make it go as fast as we want, even faster than c? What stops it from accelerating? Good questions. And there's a straightforward answer: you can't accelerate something past the speed of light because *when something goes fast, its mass increases*. The closer the speed comes to that of light, the greater its mass, and the harder it becomes to accelerate.

The way mass of an object increases with the object's velocity is shown in the graph below. We see that at a velocity of about half that of light, the mass has increased by about ten percent. When $v/c = 0.9$, the mass doubles. As the velocity approaches c, the mass goes to infinity[5]. Not that for velocities small compared to c, the increase of mass is truly tiny. Therefore what is said by the Theory of Relativity does not disagree with our everyday observations.



We must now accept the fact that mass, like velocity, is a *relative* quantity. It is different in different frames of reference. In its *own* frame of reference, the velocity of an object is, of course, zero (by definition of its *own* frame!). Likewise in its *own* frame, the frame in which it is at rest, the mass of an object always has its zero velocity value, its "rest mass". And remember, all (inertial) frames have equal status.

Is this increase of mass "real" or merely "apparent". In the next chapter, we will see that time and distance are also relative quantities, different in different frames. This change in mass is as real as the change in time and distance. And you will be able to decide for yourself on just how "real" you want to consider those differences.

5 In the next chapter, we will see this same graph for time and distance. We will then discuss how it comes about in those cases. A similar line of reasoning works to show the mass increase, but we will not go through it.

The equivalence of mass and energy

Mass was the "amount of matter". Where does the increased mass of a moving body come from? It comes from the energy you give a body by pushing on it as it moves. Ordinarily, at speeds much less than c, you give the body extra kinetic energy by increasing its velocity. But near the speed of light, when its speed cannot much increase, the energy goes into increasing its mass. Mass and energy are interchangeable.

If we have a system (a region) into or out of which no energy or mass flows from the outside, when its energy increases or decreases, for whatever reason, its mass increases or decreases by the amount

$$\Delta M = \Delta E/c^2. \qquad 9.10$$

c^2 is the conversion factor between mass and energy. One joule divided by $(3 \times 10^8)^2$ is equivalent to one kilogram.

For example, the mass of a clock *increases* when we put energy *into* it by winding it up. The increase is tiny, but it's no doubt there. The mass of a quantity of wood and oxygen *decreases* when they combine to form ash and *release heat*. The mass of the constituents of a hydrogen atom, an electron and a proton, *increases* when we put energy *into* the system to pull them apart. The mass of the fragments of a uranium atom are less after a the uranium splits apart in a fission reaction. The consequently released nuclear energy is responsible for nuclear reactors and nuclear explosions.

Only in the case of nuclear reactions is the mass change actually large enough to be readily measured. But it can be measured in chemical reactions by sensitive techniques. In every case, however, when energy is released by a fuel, the mass of the matter left over is actually less than it was to start with.

Multiplying both sides of Equation 9.10 by c^2, and leaving off the Δ's, we get

$$E = Mc^2. \qquad 9.11$$

Chapter 10

Relativity II

The new nature of time and space

What is time? If no one asks me, I know what it is. If I wish to explain it to him who asks me, I do not know.

St. Augustine

I do not define time, space, place, and motion, as being well known to all...

I. Absolute, true, and mathematical time, of itself and from its own nature, flows equably without relation to anything external...

II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable...

Isaac Newton

Subtle is the Lord...

Albert Einstein

Newton would not define time and space--everybody knew what they were, he said. But he was explicit about their being "absolute"--the same for everyone. We made the same assumption. We originally assumed that when an hour passes for the person on the moving cart, an hour also goes by for her friend on the ground. We assumed that time and space measurements were independent of the reference frame from which they were made. One person's time was the same as another's.

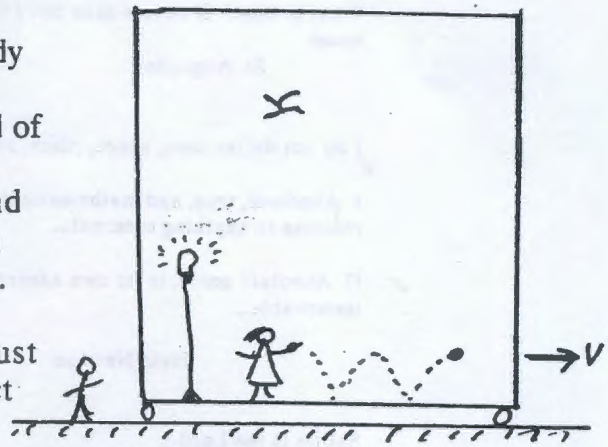
Einstein's postulate, which, by fiat, resolves the problem of the speed of light being the same for all observers challenged that assumption. We now show that it predicts a strange nature for time and space, one at variance with our earlier Newtonian assumption.

Einstein's postulate is now accepted as correct. Why? There can be only one reason: *whenever its consequences are subjected to experimental tests, they are <u>never</u> found wrong.*

Let us first explore the difference in the passage of time for observers in two different reference systems.

Time is relative

Now, with another story--a "gedanken" experiment--we use the postulate of Relativity to predict that the passage of time slows in a moving system. We then look at the experimental results confirming this effect.



<u>The fast-lady-in-a-cart story:</u> A lady in a cart travels past her stationary friend at a great rate of speed, a good fraction of the speed of light. Since her cart is moving smoothly at a constant velocity, it is a good inertial system, and the laws of nature are the same in her cart as on the ground. Everything behaves quite normally. She notes that her pulse beats about once per second, 1 hz, and balls bounce and lights flash just as they did when her cart was at rest with respect to the ground.

In fact, according to the postulates of Relativity, she can consider herself at rest, and her friend on the ground--and the ground itself--to be sliding past her at a great speed. And she adopts that perfectly legitimate attitude.

We now want to compare the time passing for her between two events in her rest frame with the time passing between the same two events for her friend on the ground.

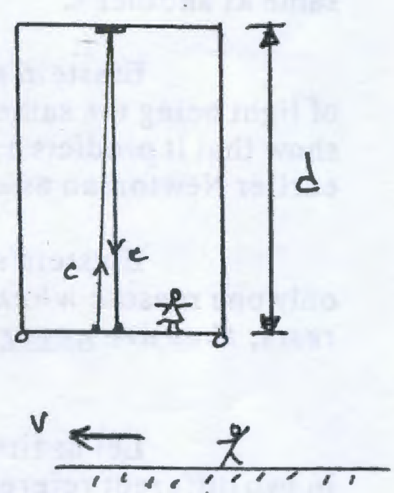


Fig 10.1

<u>In her reference system:</u> The lady will, of course, use a clock to measure the passage of time. Any good clock will do. All her good clocks agree with each other. There is, however, a clock that is readily analyzed by our techniques: a "light-clock". It allows us to make easy comparisons between two different reference systems.

With a light-clock, she can measure the time between two events by beaming a pulse of light vertically up from a flasher on the floor to a mirror on the top of her cart which then reflects it straight back down to a photocell which clicks when the light pulse hits it and immediately causes another light pulse to be sent by the flasher. The lady can now measure any longer time by just counting the clicks of her light-clock.

The time interval her clock would tick off would be just given by our old Equation 3.3, $v = \triangle x/\triangle t$, or, solving for $\triangle t$,

$\triangle t = \triangle x/v$ or, in this case,

$$\triangle t_o = 2d/c, \qquad 10.3$$

where 2d is the distance for the vertical round trip up and back from the mirror, c is the speed with which light always travels, and the time is $\triangle t_o$, where the subscript "o" emphasizes that this is the time that passed in the light-clock's <u>o</u>wn reference system, the one in which the clock is at rest.

The light-clock would, of course, agree with all the other clocks in her system: her wristwatch, her pulse, and the 1/2 inch per month rate of growth of her hair.

<u>In his reference system:</u> Let us leave the lady with her clicking light flasher and join her friend on the ground as he watches her go by. This gentleman doesn't see things quite the way she does.

In his reference system, he considers himself at rest, a perfectly valid assumption, according to the postulates of Relativity. He sees the lady moving by rapidly. According to him, her light pulse, which she (*legitimately!*) considered to go vertically up and down, does not do that at all. Her fast cart has moved the considerable distance, 2q, in the time during which the light pulse left the flasher, bounced off the ceiling mirror, and hit the photocell. The path he sees for the light is along the two diagonal lines of length p.

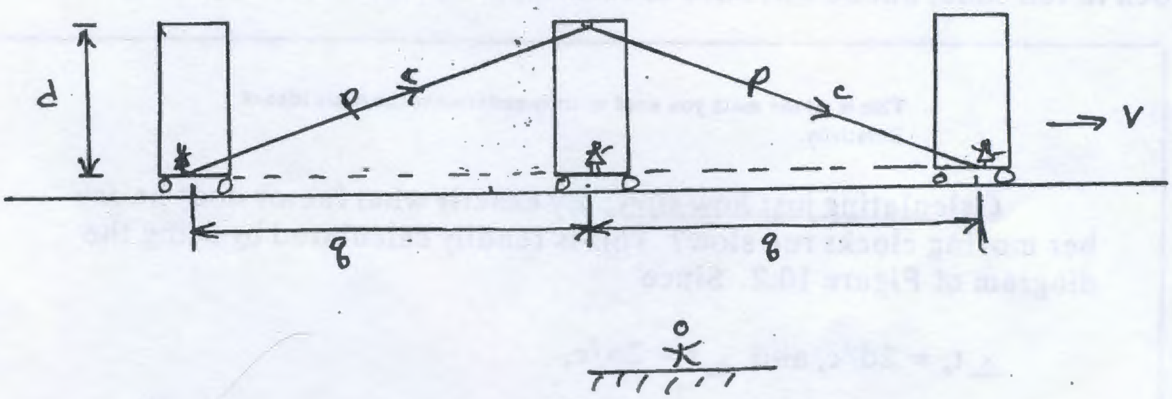


Fig 10.2

For the gentleman on the ground, the light pulse covered a greater distance between the flash and the click than that same light pulse did in her frame. Since for him the light moved at the same speed it did for her, he must say that it took a longer time to cover that longer distance. For him the time between the clicks is given by

$$\Delta t = 2p/c, \qquad 10.4$$

There is now no "o" subscript on the time because this time is *not* determined in the clock's own reference system. It is the time between the same two events (the two clicks) happening in the clock's moving system, but which are now observed in the gentleman's rest system.

Since p is greater than d, Δt is greater than Δt_o. He says a longer time has passed between clicks than she says has passed. If she looked at her wristwatch, she might say that 4 seconds passed between clicks of the photocell. She would say $\Delta t = 4$ s. If he looked at his wristwatch, he might say that 9 seconds passed between the same two clicks. He would say $\Delta t = 9$ s.

A clock that gives a smaller number of minutes, seconds, or microseconds, etc., for the time between two events is said (ungrammatically) to "run slow". The gentleman is therefore saying that the lady's light-clock is running slow. Since all the clocks in her reference frame agree with each other, her wristwatch, her hair-growth rate, etc., he is saying that all her clocks are running slow, that *time is passing more slowly in her moving reference system.*

Everything was consistent for the lady, and it is for the gentleman. All the clocks in his reference frame agree with each other, including the clock which sends a light beam along the diagonal of length p, i.e., the clock physically in the moving system, but as interpreted by him. He can use this moving light-clock to tell time, but he will have to correct for the fact that it runs slow.

This is all the math you need to truly understand the basic idea of Relativity.

Calculating just how slow: By exactly what factor does he say her moving clocks run slow? This is readily calculated by using the diagram of Figure 10.2. Since

$$\Delta t_o = 2d/c, \text{ and } \Delta t = 2p/c,$$

$$\Delta t = (p/d)\Delta t_o.$$

Noting from Figure 10.2 that 2q and the distances d and p are related to each other by the Pythagorean Theorem for right triangles

$$\rho^2 = d^2 + q^2 \quad \text{and therefore} \quad d = \sqrt{\rho^2 - q^2}$$

Therefore, rewriting the relation of Δt and Δt_o,

$$\Delta t = \frac{\rho}{d}\Delta t_o = \frac{\rho}{\sqrt{\rho^2 - q^2}}\Delta t_o = \frac{1}{\sqrt{1 - q^2/\rho^2}}\Delta t_o$$

But $q = v\Delta t/2$, and $p = c\Delta t/2$. Therefore, substituting these in above,

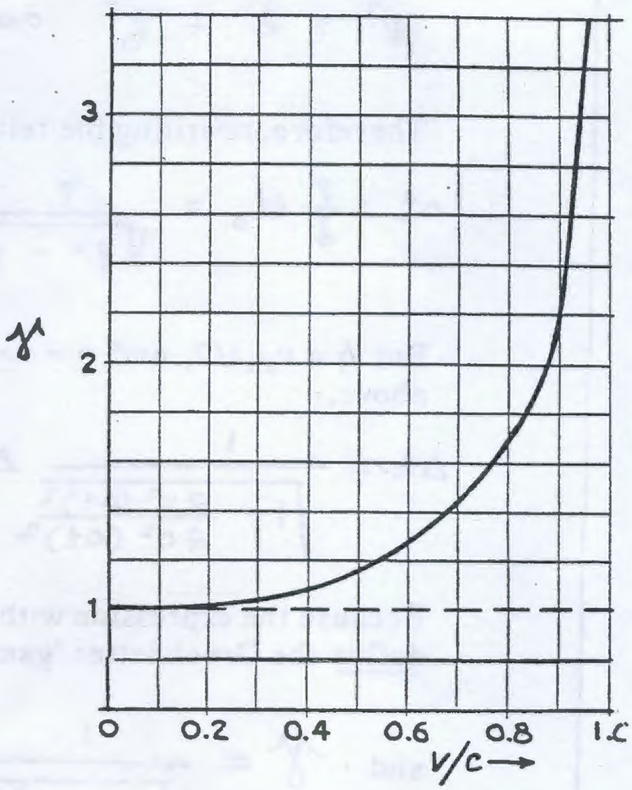
$$\Delta t = \frac{1}{\sqrt{1 - \frac{2v^2(\Delta t)^2}{2c^2(\Delta t)^2}}}\Delta t_o = \frac{1}{\sqrt{1 - v^2/c^2}}\Delta t_o$$

Because the expression with the square root comes up so often, we define the Greek letter "gamma"

and $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (10.5) \quad \text{and} \quad \boxed{\Delta t = \gamma \Delta t_o} \quad (10.6)$$

This is the relation between the amount of time that passes in a moving system and that passing in a system at rest. These two *different* times are the times between the *same* two events. By how much do these times actually differ?

In Figure 10.3 we plot γ as a function of v/c. Note that γ is always larger than one, but that it differs extremely little from one except when the velocity of the moving system becomes comparable to the speed of light. Note particularly that as v approaches c, γ becomes infinitely large.



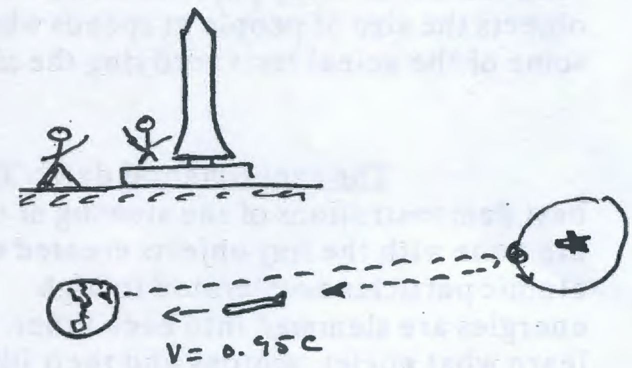
What if the gentleman looked at the lady's wristwatch? Would he actually see it running slow? Yes. But we must be careful about what we mean by "seeing". The light coming to his eye from different parts of the watch must travel by different paths, and when these paths are changing their length rapidly, we have a complicated situation. The image falling on his retina at any time is not a good representation of the moving object. Instead of talking of "seeing", we should always speak of "measuring". A good measurement will show the wristwatch to be running slow. We will actually continue to use the word "see", but in its general sense of "understand". By his measurements he will understand all her clocks to run slow.

<u>Symmetry:</u> We have used the postulate of Relativity to calculate exactly how much slower clocks run, and time passes, in moving systems than in the one at rest. But according to the postulate, *either* system can be considered at rest. What we calculated was the slowing of her time as seen by him. She has every bit as much right to consider herself at rest. She could look at a light-clock he builds in his system and decide that *his time was running slower than hers.*

<u>Real or apparent slowing?:</u> This symmetry is fine and just what is required by the postulate of Relativity. But if this is so, are we not just talking about an *apparent* slowing of clocks in moving systems? How can she see his clocks run slow and he see hers run slow and have this be a "real" effect.

As a matter of fact, a gedanken experiment was concocted shortly after Einstein's formulation of Relativity
to show that the slowing of moving clocks was illusory--an artifact of the observation process. The story is called "The Twin Paradox", and it goes like this.

One of a pair of 25 year-old twins takes off on a fast rocketship for a visit to a planet of a distant star. He rapidly accelerates to a speed of 0.95c and stays at that speed until he reaches the star. Once there, he spends very little time before reembarking and returning home, again at the speed of 0.95c.



During the astronaut's travels 60 years pass for the twin on earth. A stooped and grey man of 85 comes to the rocketship landing site to greet his astronaut brother, who spryly jumps from the landing pad. After all, for the twin travelling at 0.95c, $\Delta t_o = \Delta t/\gamma$. From our graph, we can see that for $v/c = 0.95$, $\gamma = 3$. Only (1/3)(60) = 20 years have passed for the astronaut. He is now only 45 years old.

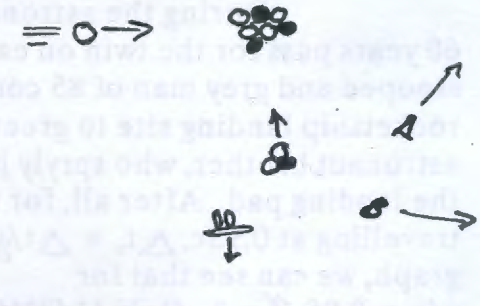
That is surely what Relativity says. But now the "paradox". Relativity has a symmetry. Should we not be able to say that the astronaut never moved. It was rather his earth-bound brother and the rest of the universe, including the distant star, which was the moving system. In that case, the astronaut should be 85 and the stay-at-home twin only 45. How then can this difference in the passage of time be real? Relativity seems to pose a paradox and is therefore in trouble.

The paradox is phony. There is actually no symmetry in this case. Relativity told us that the laws of nature are the same in all *constant velocity* (or "inertial") systems. We derived the relativistic equations for constant velocity systems. Only the inertial stay-at-home twin may apply these relations. His conclusion, that he aged much more than his twin, is valid. Application of them by the twin who accelerated is invalid.

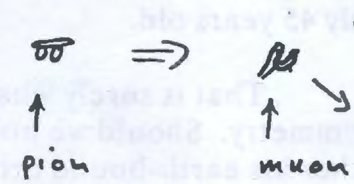
That there is no symmetry here is demonstrated by the fact that the accelerating twin could tell he was accelerating. He could, for example, feel the force of the seat back accelerating him. *During his long periods of constant velocity travel*, he could legitimately consider his brother's clocks running slow. But *during* his accelerations, which were very large if he changed his velocity by a lot in a short time, he would conclude that time was passing not slow but extremely *fast* for his brother. The details of the accelerating system are complex and we will not discuss it.

So is the slowing of clocks real? Yes. One object can age more rapidly than another. Relativity says that you can, in principle, become older than your mother in every physical sense. Of course, we have not (yet?) moved any objects the size of people at speeds where the effects are large. Let's talk about some of the actual tests verifying the slowing of moving clocks.

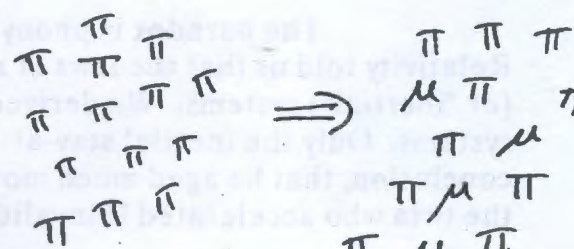
The experimental data: The best demonstrations of the slowing of time are done with the tiny objects created when atomic particles accelerated to high energies are slammed into each other. (We learn what nuclei, protons and their ilk are made of by seeing what flies out in such collisions.) One type of object formed in this way is the "pion".



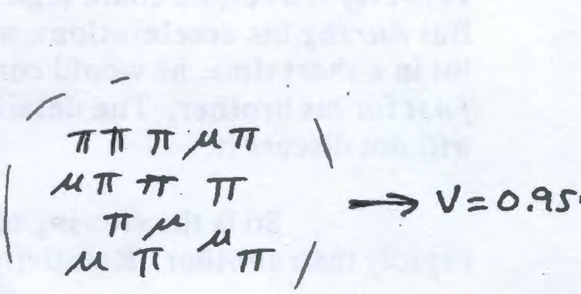
Pions: One of the best examples of clock slowing is an observation done on pions. Pions are unstable, and rapidly change, or "decay" into another kind of particle called a "muon". Each pion decays randomly. That is, if you start out with a large group of pions, at the end of a time called the pion "half-life", one-half of the group of pions will have turned into muons.



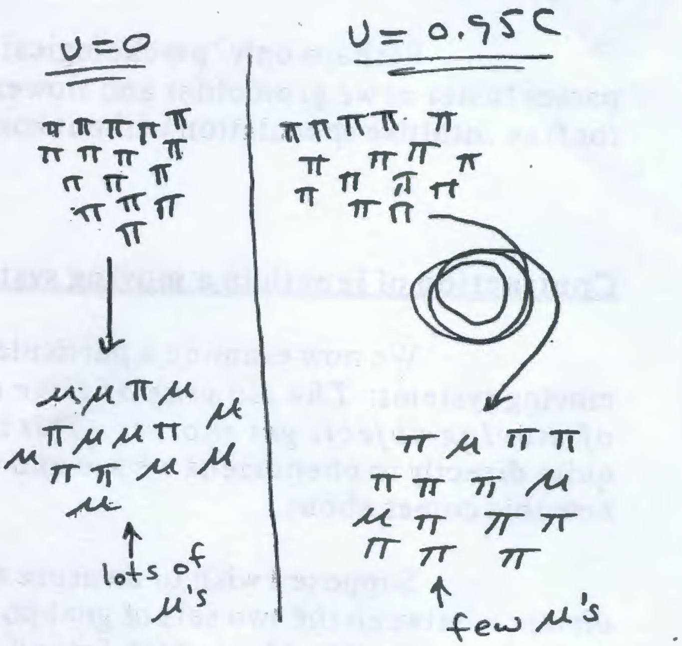
Therefore, if we start out with a group of pions, we can tell how much time has passed for them by noting what fraction of them has become muons. We can tell the "age" of a group of pions by seeing what fraction of the group has become muons.



It is possible to accelerate a group of pions to speeds very close to that of light. One finds that a fast moving group of pions turns into muons at a much slower rate than a group of pions which are stationary in our reference system. The slowing is exactly that predicted by Relativity theory.

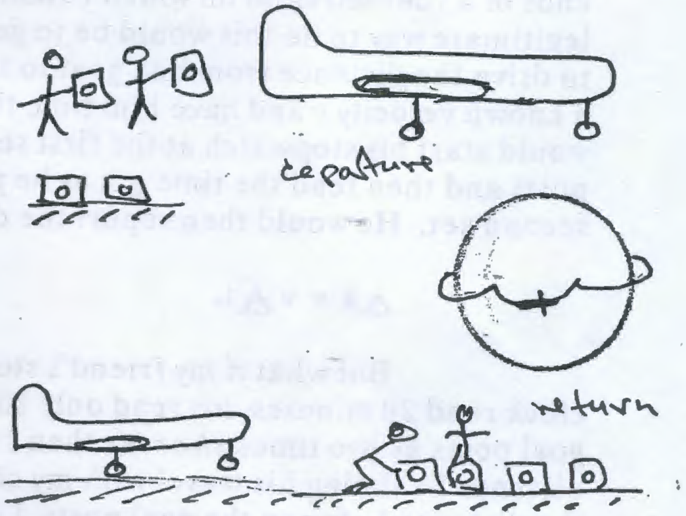


We can actually do the "twin paradox" experiment with pions. Two groups of pions of identical "age", can be created. One may be left at rest in the laboratory and the other accelerated to extremely high speeds traversing a circular orbit many times. When the speeding group of pions is brought to rest, it can be compared to its "twin" which never moved. Since time passed more slowly for the moving pions, fewer of that group will be found to have become muons. The group which traveled at great speed will be much younger.



Clock on a fast jet: Perhaps the most dramatic demonstration of relativistic slowing of time was performed by physicists who built some extremely accurate clocks. To test Einstein's theory, they left some of their clocks in Washington, D.C., and took others around the world on commercial jets flying about five hundred miles per hour and actually stopping over in various cities.

At such slow speeds, the relativistic slowing of a clock is only a very small fraction of a second, even for a round-the-world trip. But the clocks were highly accurate. When the researchers got back to Washington, and compared the moving clocks with the stay-at-home clocks, the travellers were slow by just the predicted amount.



There have been a vast number of indirect experiments where the relativistic slowing of time is significant. *In no case has the prediction of the Theory of Relativity ever been shown wrong.*

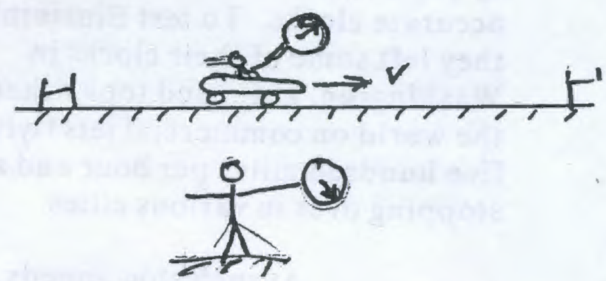
A comment on thinking deeply on the nature of time: In times past, philosophers speculated freely on the deeper nature of time. (I'm not sure if any ever seriously proposed anything similar to, or as strange as, what we now know to be true. But that's not the point I wish to make.) Today, if you wish to speculate on the deeper nature of time, you *must* start out with what we know from Relativity. That's true, at least, if you are talking of *physical* time, that wristwatches, the moving planets, and the rate of growth of hair measure. That should make any speculation particularly exciting. You start out with a fantastic perspective.

Perhaps only "psychological time", or perceived time--the kind that passes faster as we grow older and slower when we are bored--remains available for free intuitive speculation without consideration of relativity.

Contraction of length in a moving system

We now examine a particular consequence of the slowing of time in moving systems: *The slowing of time in moving systems tells us that lengths of moving objects get shorter.* This relativistic contraction actually leads quite directly to phenomena we see and use in daily life. Let's try to understand how this comes about.

Suppose I wish to measure the distance between the two sets of goal posts at the ends of a football field on which I stand. One legitimate way to do this would be to get a friend to drive the distance from one goal to the other at a known velocity v and have him time the trip. He would start his stopwatch at the first set of goal posts and then read the time Δt as he passed the second set. He would then report the distance as

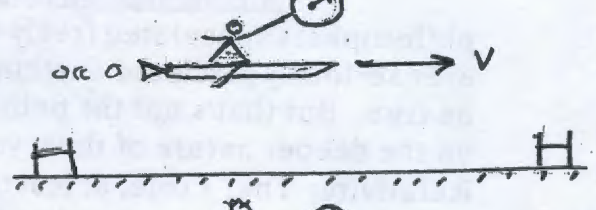


$$\Delta x = v \Delta t.$$

But what if my friend's stopwatch ran slow? Say that when my *good* clock read 20 minutes, his read only 10. He would report the distance between the goal posts as two times *shorter* than I would say it was. I could determine that distance by timing his travel with my clock. Or, since I am at rest in the frame of the distance between the goal posts, I could also have measured the distance with a meter stick. Both my methods would, of course, agree.

With this simple example, we have almost made our point. If someone measures a length from a moving system this way, but uses a slow clock, they will find a shorter length.

Now, we have previously shown that clocks do indeed run slow in a moving system, by an appreciable amount for velocities close to that of light. If someone zipped by on a fast rocket and measured the distance between the goal posts by knowing their speed and timing their trip, even if



they used perfectly good clocks, they would measure a shorter distance. (I would, of course, say their clocks were running slow.)

In the reference frame of the rocket, as valid a frame as mine, the football field is moving past them at a great rate. They would measure ("see") contracted lengths for football fields, or anything else, moving past them. The faster they it moved by, the shorter it would become. This holds not just when the lengths are measured by timing. In their perfectly valid frame of reference all measurements of the length of things must agree.

The shortening of moving objects[1] is by just the factor that moving clocks run slow.

$$\Delta t = \gamma \Delta t_0 \qquad \Delta t_0 = \Delta t / \gamma$$

$$\text{and } l = l_0 / \gamma \qquad \gamma \text{ always} > 1$$

where and are the distance and time as seen in the not-moving system and *is the distance in the rest frame of the length*, the length's own frame, the *moving* system.

Careful. The equations for time and length are not symmetrical. Just remember three things: 1) that the factor relates the moving lengths and times, 2) that a moving clock runs slow and therefore reads a shorter time (Δt_0), and 3) a moving length (l) contracts and is a shorter distance.

Do things *really* get shorter when they move? As in the story of time, relativity forces us to carefully define what we mean by "length". Just "looking" at the length of an object is quite meaningless when that object moves by us at a speed close to that of light. Light from the different ends of the object travel to the eye by different paths, and such light getting to the eye at the same time does not necessarily leave the object at the same time. "...the same time.." is itself something we have to be very careful about. All this makes "looking" a complex process.

But if we define length in the only meaningful way, as the result of a measurement process, the one my friend used in measuring the football field for example, the shortening predicted by Relativity is indeed real. It is of course the

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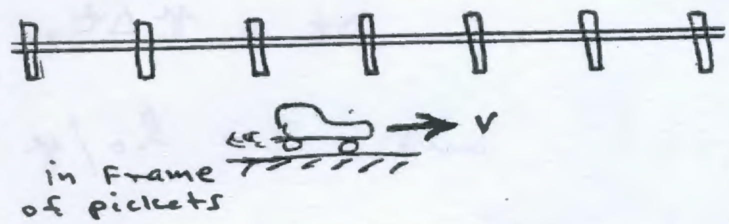
[1] The shortening is in the direction of motion only. It's the only direction to which our original example applies. The driver does not conclude, for example, that the distance on the road between his two front wheels changes even if his clock is slow.

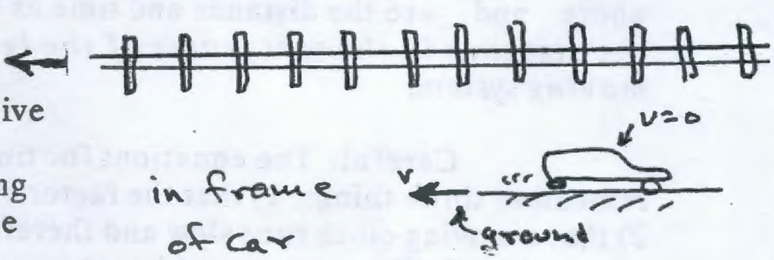
length of the moving object in the "rest frame" which is shorter. In its own frame, the length is, of course, always at rest, and its length, its "proper length" does not change.

As with time, instead of talking of "seeing" a moving length as contracted, we should speak of "measuring" a moving length as contracted. But we will continue to use "see" in the more general sense of the word where it means "understand". The length contraction effect is well demonstrated, and we discuss an important example of that now.

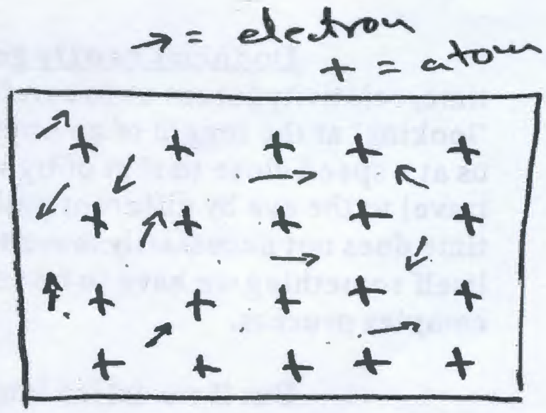
Relativistic forces between moving currents (no math, but tricky!)



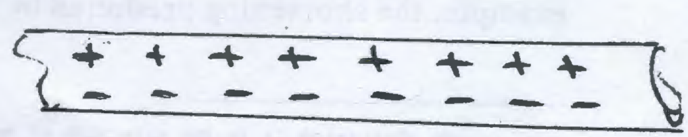
If you move rapidly past a picket fence, in your frame, the spacing between the pickets contracts according to our relativistic equation. With the pickets closer together, you see more pickets in your neighborhood. This effect is true for the distances between any objects.



The objects we want to talk about are the two kinds of charges in a conducting wire. They are the free negative electrons moving in the wire and the positively charge atoms of the wire making up the bulk of the metal. (Remember, the atoms of a metal are positively charged because they have each lost an electron to the free electron crowd.) The positive atoms in the metal are more or less equally spaced in three dimensions, but thinking of them as being in a single straight line is a good enough model for us now. The electrons in a copper wire are not stationary. But in their random motion, they maintain some average distance. Thinking of them in a single straight line and neglecting their random motion is a fine simple picture.

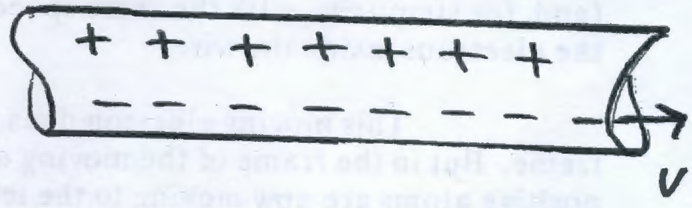


In a metal there are just as many free electrons as there are positively charged atoms. We represent a short section of a very long straight wire in Figure 10.2a. With the same number of free



electrons and atoms in each meter, the wire is electrically neutral, uncharged.

Now let a current flow. Our story gets a bit complicated. One effect will causing another, and we will need to look at things from different reference frames. But let's plunge on.

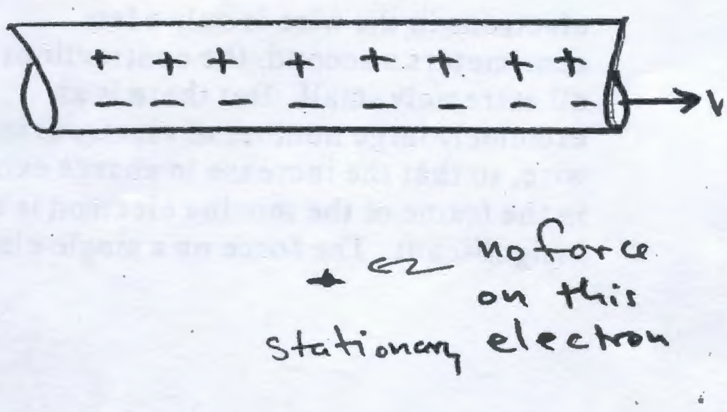


As an electric current flows in our wire, the free electrons move to the right with velocity "v". The positive atoms, and thus the wire itself, remain fixed. Although v is very small, there is nevertheless *some* extremely tiny relativistic contraction of the average space between the electrons because of their motion. In the rest frame of the wire, the electrons become closer together. It might seem that there would thus be more negative electrons than positively charged atoms in a section of wire, and that section would be negatively charged. But this does *not* happen.

Why not? (And how do we know?)

The "Why not?" first. The relativistic contraction does in fact bring the electrons closer to each other in the rest frame of the atoms. But the atoms, being electrically attracted to the electrons, squeeze together to stay as close as possible to their electrons.

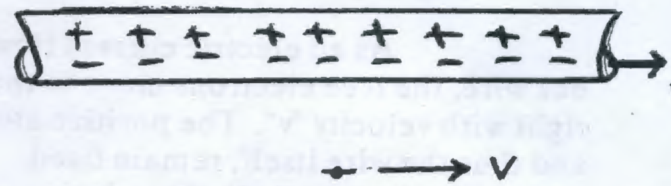
When the electrons start to flow they get closer together and actually and pull the atoms closer to each other with them. The whole wire gets a very tiny bit shorter, but it remains uncharged. Since typical electron speeds in carrying a current are only a few centimeters a second, the change in length of a meter of wire is a tiny fraction of a single atom. The shortening is insignificant. The important point is that even though the electrons must get closer together because of their motion, the wire stays electrically neutral even when it carries a current.



How do we know this is true? How do we know the wire stays electrically neutral? If a current-carrying wire were not neutral, an electron at rest outside it would be attracted or repelled. This does not happen. there is no electric field generated toward or away from a wire when it carries a current. If the atoms did not squeeze down to join their slightly closer together electrons, there would be such a field and a resulting force on an external charge.

The stationary outside electron therefore sees an equal number of stationary atoms and moving electrons in each section of wire.

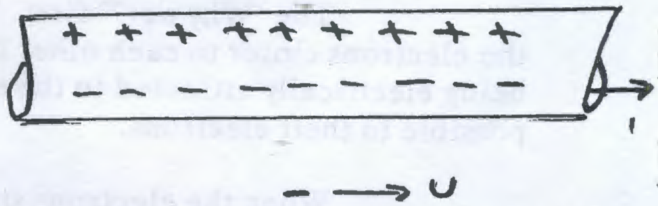
But now suppose our outside electron were to move in the same direction (and, for simplicity, with the same speed) as the electrons inside the wire.



This moving electron does not change the wire in the wire's own frame. But in the frame of the moving outside electron, the previously stationary positive atoms are now moving to the left with velocity v, and the space between them therefore relativistically contracts. The previously moving electrons in the wire are now stationary. The previously relativistically contracted distances between them "uncontracts", expands. The moving outside electron therefore sees an increase in the positive charge in each length of wire and a decrease of negative.

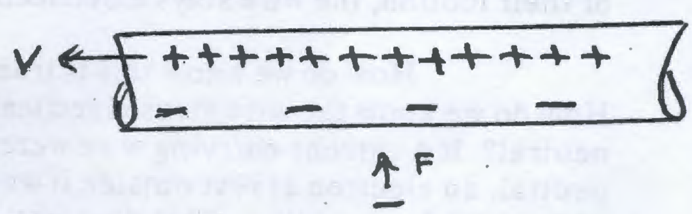
The positive charges closer together and the negatives farther apart than in the stationary frame of the wire--in which it was neutral--both make the current carrying wire have a net positive charge in the frame of the outside electron.

The conclusion is that while a stationary electron outside a current carrying wire sees that wire as neutral and feels no force, when the electron outside the current carrying wire moves in the direction of the wire electrons, it sees the wire as positively charged. The outside electron will therefore experience an electrical force and be attracted to the wire. This force is due to relativistic length contractions.



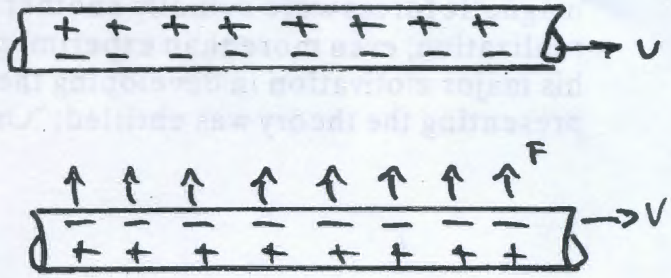
Picture in wire's frame (wire at rest)

Since the velocity v of the electrons in the wire is only a few centimeters a second, the contractions are all extremely small. But there is an extremely large number of electrons in the wire, so that the increase in charge existing in the frame of the moving electron is not insignificant. The force on a single electron

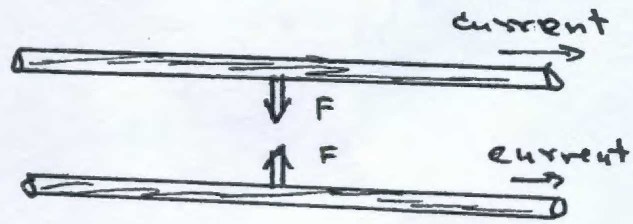


Picture in electron's frame (electron at rest) due to a current in a nearby wire can easily be enough to substantially deflect the motion of this tiny mass.

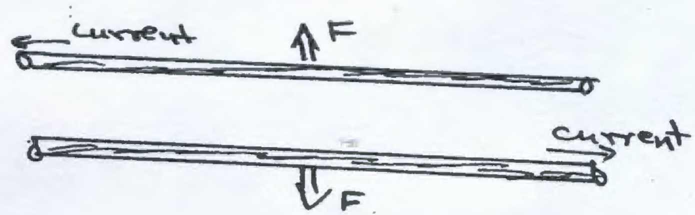
Suppose now our "outside electron" is not alone. Let it be inside a second wire with a vast number of companions all moving along with it carrying a current in that second wire. This electron and each of its companions will feel the same force pulling them toward the first wire. The total force on this large number of electrons is not necessarily small.



By Newton's Third Law, there will be an equal and opposite force on the first wire by the second. *Two wires carrying current in the same direction will attract each other.*

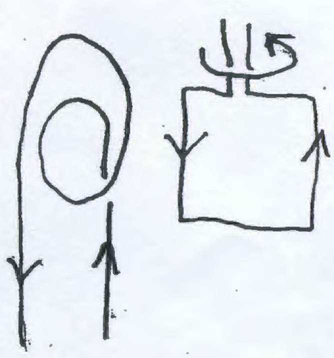


One can go through this whole line of reasoning for the current in the two wires in the opposite directions. One would show that *two wires carrying current in opposite directions repel each other.*



These relativistic forces can actually be very large. When power companies have two parallel wires carrying large currents in opposite direction near each other, the wires must be held together by heavy steel bands.

Loops of wire carrying current can be arranged so that strong forces arise to make the loops rotate. That's how electric motors work. Our vacuum cleaners and the heavy motors in factories work by this relativistic effect.



These forces were, of course, known long before Einstein's Relativity. They are generally called "magnetic forces". In the 19th Century they were recognized as a force in addition to the ordinary electric forces between charges arising from the motion of charge. What Einstein realized was that these magnetic forces were actually another aspect of the electric force. This realization, even more than experiments such as Michaelson and Morely's, were his major motivation in developing the Theory of Relativity. The 1905 paper presenting the theory was entitled: "On the Electrodynamics of Moving Bodies".